I-CELMECH
TRAINING SCHOOL

February 3-7, 2020

Location
Department of Mathematics
"Federigo Enriques" - University of Milan

Invited lectures and lecturers:

Space debris
Catalin Gales, University of Iasi

The n-body problem
Gabriella Pinzari, University of Padova

Tidal Effects and Rotation of Extended Bodies
Gwenaël Boué, IMCCE Paris

Orbital determination with the Kepler Integrals
Giovanni Federico Gronchi, University of Pisa

Nekhoroshev theory and Arnold diffusion
Christos Efthymiopoulos, Academy of Athens

Superintegrable systems and their perturbations
Francesco Fassò, University of Padova

Introduction to KAM theory and applications
Ugo Locatelli, University of Rome "Tor Vergata"

Dissipative KAM theory
Alessandra Celletti, University of Rome "Tor Vergata"

Theory and applications of FLI for the computation of transit orbits
Massimiliano Guzzo, University of Padova

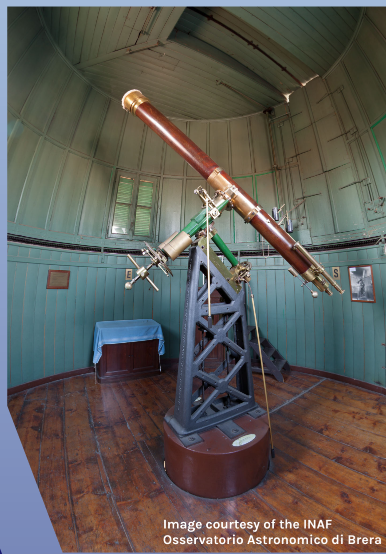
Image courtesy of the INAF Osservatorio Astronomico di Brera

Scientific Committee

Giulio Baù
Alessandra Celletti
Antonio Giorgilli
Massimiliano Guzzo
Tiziano Penati
Marco Sansottera

Organizing Committee

Veronica Danesi
Beatrice Langella
Tiziano Penati
Marco Sansottera

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI MATEMATICA "FEDERIGO ENRIQUES"

PROJECT MIUR-PRIN 20178CJA2B
New frontiers of Celestial Mechanics: Theory and Applications

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About

I-CELMECH

PROJECT MIUR-PRIN 20178CJA2B

"New frontiers of Celestial Mechanics: theory and applications"

The project I-CELMECH focuses on mathematical methods in Celestial Mechanics with the goal of advancing in parallel on the theoretical part (perturbative and stability theories) and to provide applications to astrodynamical problems of interest for space agencies and industries.

Several members of the project are recognized leaders in perturbation methods; to plan a further jump we identified theoretical research areas (explicit normal form construction; KAM-Nekhoroshev theories; regularizations; non Hamiltonian models), whose rigorous methods can provide powerful tools to applied problems (space debris; extra-solar planets; low-energy orbits in Astrodynamics; orbit determination of NEO; geodynamics). Our objectives combine theory and applications focusing on: normal forms for space debris; KAM tori and resonances in extrasolar systems; space manifold dynamics; gravitating systems with noise; patched dynamics for orbit determination; geodynamics of Jupiter satellites; non Hamiltonian stability. I-CELMECH aims at reaching different goals: an innovative research program, a high-level training, new collaborations also with space agencies/industries.

I-CELMECH has potential continuation in European excellence H2020 and with the foundation of an Italian Mathematical Space Center.

More info at `http://www.mat.unimi.it/I-CELMECH/`

I-CELMECH Training School

The I-CELMECH Training School will take place at the Department of Mathematics "Federigo Enriques" of the University of Milan.

The aim of the Training School is to present a contemporary review of recent results in the field of Celestial Mechanics.

Special emphasis will be placed on the theoretical aspects.

The Training School will consist of 9 courses and a limited number of contributed talks.

Scientific Organizing committee

Giulio Baù (University of Pisa)

Alessandra Celletti (University of Rome "Tor Vergata")

Antonio Giorgilli (University of Milan)

Massimiliano Guzzo (University of Padova)

Tiziano Penati (University of Milan)

Marco Sansottera (University of Milan)

Local Organizing committee

Veronica Danesi (University of Milan)

Beatrice Langella (University of Milan)

Tiziano Penati (University of Milan)

Marco Sansottera (University of Milan)

Timetable

KL: Keynote Lecture, **CT**: Contributed Talk.

Remark. Lunch is not served by the organization of the school. A list of possible restaurants, coffee bars, take aways in the area around the University is provided in the "Useful Information" section.

Monday (February 3, 2020)

08:30–09:30	**Registration**		
09:30–10:00	**Opening**		
10:00–12:00	**KL**	**U. Locatelli** UniTOV	Introduction to KAM theory and applications
12:00–14:00	**Lunch**		
14:00–16:00	**KL**	**G. Pinzari** UniPD	The n-body problem
16:00–16:30	**Coffee Break**		
16:30–16:45	**CT**	**I. Gkolias** PoliMI	An analytical insight into the end-of-life disposal of close-Earth satellites using a solar sail
16:45–17:00	**CT**	**D. Menzio** PoliMI	An application of the Flyby map to Europa Clipper mission
17:00–17:30	**CT**	**C. Colombo** PoliMI	COMPASS project for space debris modelling and mitigation

Tuesday (February 4, 2020)

09:00–11:00	KL	**A. Celletti** UniTOV	Dissipative KAM theory
11:00–11:30		**Coffee Break**	
11:30–11:45	CT	**R.I. Páez** UniPD	Study of large Lyapunov orbits and temporary captures via the Levi-Civita Hamiltonian normalization
11:45–12:00	CT	**R. Goncalves Schaefer** UPC	Arnold diffusion for a Hamiltonian with 3+1/2 degrees of freedom
12:00–12:15	CT	**F. Mogavero** IMCCE	Origin of Chaos and Resonant Structure in the Inner Solar System
12:15–12:30	CT	**N.H. Hoàng** IMCCE	Long-term destabilisation of Mercury orbit: the dynamical mechanism
12:30–12:45	CT	**M. Pellegrino** CU	Utilizing Chaos for Debris Mitigation
12:45–13:00	CT	**I. Milić Žitnik** AOB	The specific property of motion of resonant asteroids with very slow Yarkovsky drift speeds
13:00–14:30		**Lunch**	
14:30–14:45	CT	**G. Pichierri** MPIA	The onset of instability in resonant chains
14:45–15:00	CT	**S. Di Ruzza** UniPD	Numerical evidence of symbolic dynamics in a three-body problem
15:00–15:15	CT	**G. Duarte** UB	Invariant Manifolds near L_1 and L_2 in the Planar Elliptic Restricted Three-Body Problem
15:15–15:30	CT	**L. Bernus** IMCCE	Constraining the Mass of the Graviton with the Planetary Ephemeris INPOP
15:30–15:45	CT	**O. Fuentes-Munoz** CU	Extremely long-term asteroid propagation
15:45–16:15		**Coffee Break**	
16:15–16:30	CT	**J. Daquin** UniPD	Manifold structure and chaotic ballet of navigation satellites
16:30–16:45	CT	**J. Gimeno** UB	State-dependent perturbation of planar ODEs
16:45–17:00	CT	**J. Cardoso dos Santos** ASRI	Analysis of parameters space for intermediaries of the roto-orbital problem
17:00–17:15	CT	**L. Ruiz dos Santos** UNIFEI	Modeling of three-dimensional coupling of spin, orbit and tides by the Lagrangian formalism
17:15–17:30	CT	**Y. Gevorgyan** IME	Andrade rheology in time-domain. Application to Enceladus' dissipation of energy due to forced libration

Wednesday (February 5, 2020)

09:00–11:00	KL	**G. Boué** IMCCE	Tidal Effects and Rotation of Extended Bodies
11:00–11:30	**Coffee Break**		
11:30–13:30	KL	**C. Efthymiopoulos** UniPD	Nekhoroshev theory and Arnold diffusion
13:30–14:30	**Lunch**		
14:30–16:00	**Visit to the Museo Astronomico di Brera (GROUP 1)**		
16:00–17:30	**Visit to the Museo Astronomico di Brera (GROUP 2)**		

Thursday (February 6, 2020)

09:00–11:00	KL	**G.F. Gronchi** UniPI	Orbital determination with the Keplerian Integrals
11:00–11:30	**Coffee Break**		
11:30–13:30	KL	**C. Gales** Iasi	Space debris
13:30–15:00	**Lunch**		
15:00–17:00	KL	**F. Fassò** UniPD	Superintegrable systems and their perturbations
17:00–17:30	**Coffee Break & Discussions**		
17:30–22:00	**Meeting restricted to PRIN Members**		

Friday (February 7, 2020)

09:00–11:00	KL	**M. Guzzo** UniPD	Theory and applications of FLI for the computation of transit orbits
11:00–12:30	KL	**A. Giorgilli** UniMi	The unaccomplished perfection of Kepler's world
12:30–13:00	**Closing**		
13:00–14:00	**Lunch**		

List of Abstracts

Keynote Lectures

Tidal Effects and Rotation of Extended Bodies

G. Boué

KL

IMCCE, Paris, France

Accurate models of the rotation motion and/or the deformation of extended bodies allow to make prediction on their long term evolution but also to get some information about their internal structure. This is particularly true for solar system objects, but also for exoplanets. Indeed, after the discovery of thousands of extrasolar planets, the field is moving from detection to characterisation which raises questions about their rotation states and the amount of energy they dissipate by tides.
In this lecture, both aspects of rotation and tidal deformation are addressed. In a first part, I introduce Poincaré's equations of motion for a rotating mechanical system described by a Lagrangian or a Hamiltonian. In the second, I present the physical concept of tides and its different modelisations in the literature. As a running illustrative example, the case of an extended satellite orbiting a point mass planet is considered.

Dissipative KAM theory

A. Celletti

KL

University of Rome Tor Vergata, Rome, Italy

We will give an overview of KAM theory in some dissipative systems which might have applications to Celestial Mechanics. Precisely, we consider conformally symplectic systems, namely systems whose evolution in time transforms a symplectic form into a multiple of itself.

We present results on the existence of quasi-periodic attractors for conformally symplectic systems in non-perturbative regimes. For such systems, finding the solution requires also to add a *drift parameter*. We provide a very explicit quantitative theorem in an a-posteriori format, namely assuming the existence of an approximate solution, which satisfies an invariance equation up to an error term which is small enough with respect to explicit condition numbers, then we can state the existence of a solution nearby.

The theorem provides a very efficient algorithm to generate the solution, which can be implemented successfully in model problems and physically meaningful examples.

The content of these lectures refers mainly to works in collaboration with R. Calleja and R. de la Llave.

Nekhoroshev theory and Arnold diffusion

C. Efthymiopoulos

KL

University of Padova, Padua, Italy

Starting with Arnold's pioneering work in 1964, the term "Arnold diffusion" refers to the slow diffusion taking place along resonances in Hamiltonian nonlinear dynamical systems with three or more degrees of freedom. The course will introduce basic concepts related to our current understanding of the mechanisms leading to Arnold diffusion, with emphasis on phase space structures (e.g. the resonant manifolds and the stable and unstable manifolds emanating from several classes of hyperbolic invariant objects residing in phase space) which drive slow chaotic diffusion. A precise quantification of the speed of Arnold diffusion can be done using methods of canonical perturbation theory, i.e., with the construction of a Nekhoroshev normal form and the study of the dynamics induced by the latter's optimal remainder. In particular, the course will refer to methods allowing to arrive at accurate estimates on the diffusion rate using Melnikov and stationary-phase methods. Applications of the above in systems of concrete physical interest are based on computer-algebraic implementations of normal form algorithms. We will enumerate a number of problems in Celestial Mechanics and Astrodynamics where such approaches lead to practically useful results.

Superintegrable systems and their perturbations

F. Fassò

KL

University of Padova, Padua, Italy

Most of the integrable systems of interest in Celestial Mechanics (e.g. resonant harmonic oscillators, the Kepler system, the Euler-Poisson rigid body) are “superintegrable”: due to the existence of many integrals of motions, or of a large symmetry group, their motions are linear on tori of dimension smaller than the number of degrees of freedom. The study of perturbations of these systems, particularly in the realm of Nekhoroshev theory, requires a comprehension of their geometric and dynamical structure.
The description of superintegrable systems was initiated by Nekhoroshev and was later developed by Mishenko and Fomenko (“noncommutative integrability”) and others. This description goes beyond the one, often considered to be the utimate one for Hamiltonian systems, provided by the Liouville-Arnold theory. While Liouville integrability focuses on the existence of a single foliation by (Lagrangian) invariant tori, the natural object to look at in superintegrable systems is a double foliated structure. The finest foliation is formed by the (isotropic) low-dimensional invariant tori. The leaves of the coarsest foliation are formed by certain (coisotropic) families of invariant tori that carry motions with the same frequencies.
Mathematically, the leaves of the finest foliation are the common level sets of all the integrals of motion or, equivalently, orbits of maximal compact abelian subgroups of the symmetry group. The leaves of the coarser are the common level sets of the (true) actions or, equivalently, orbits of the entire symmetry group. The two foliations form what in symplectic geometry is called a “dual pair”. Pictorially, they can be figured out as formed by a sets of “flowers” decomposed into their “petals”.
This double structure is basic for perturbation theory: the normal forms of perturbation theory must and can be built not in neigbourhoods of individual invariant tori/petals, but in neighbourhoods of entire flowers. The (symplectic) geometry of this structure allows to prove that this is indeed possible. This provides a clear understanding of Nekhoroshev theorem for superintegrable systems and its dynamical consequences.
The talk will provide a mathematically friendly introduction to this topic. For a review, see the article *Superintegrable Hamiltonian systems: geometry and perturbations* by the author (Acta Applicandae Mathematicae **87**, 93-121 (2005)).

Space debris

C. Gales

KL

University of Iasi, Iași, Romania

The intense exploitation of circumterrestrial space generates inevitably a lot of space debris, namely a huge collection of uncontrolled human-made objects in orbit around the Earth. Space debris include spent rocket bodies and non-operative satellites, but also fragments generated by destructive events (such as collisions and explosions). Modelling various perturbations, studying the effects of resonances, determining the equilibrium points and the stable or chaotic regimes are nowadays mandatory. Such information is crucial for defining post mission disposal solutions and for designing possible removal techniques for existing debris. Over the last few years, various mathematical models and approaches have been developed to investigate the long-term dynamics around the Earth. Such models depend on the explored region and the types of objects, since the strength of the different forces (geopotential, atmospheric drag, lunar and solar attractions, solar radiation pressure, etc.) varies with the altitude and parameters of debris. We present several recent approaches and results describing the long-term dynamics of space debris in different circumterrestrial regions. In particular, we discuss the effects of various perturbing forces, the effects of resonances and the occurrence of some dynamical phenomena, like bifurcation of equilibria, chaos, interplay between conservative and dissipative forces, which lead to complex evolutions of the orbital elements of space debris.

This talk refers to several works in collaboration with Alessandra Celletti.

The unaccomplished perfection of Kepler's world

A. Giorgilli

KL

University of Milan, Milan, Italy

A short trip trough the main works of Kepler, notably the *Astronomia Nova*, trying to follow his search of the perfection of the World till the discovery of his celebrated laws. At the end of the road, the consciousness that the finish line had not yet been reached.

Orbit determination with the Keplerian Integrals

G. F. Gronchi

KL

University of Pisa, Pisa, Italy

In the last years the improvements in the observational technologies produced new mathematical problems in Orbit Determination (OD). One of them is the linkage problem, occurring with the large amount of optical observations collected with modern telescopes: we have to identify Very Short Arcs (VSAs) of asteroid observations made in different nights as belonging to the same celestial body. In this lecture we shall review some modern OD methods, introduced to deal with the linkage problem. All these methods employ the first integrals of Kepler's motion and admit a polynomial formulation. Being based on conservation laws, they can be used with pairs of VSAs with a large time separation, even more than one period of the asteroid orbit, as opposite to the classical OD methods by Gauss and Laplace.

Theory and applications of Fast Lyapunov Indicators for the computation of transit orbits

M. Guzzo

KL

University of Padova, Padua, Italy

Chaos indicators have been used in the literature to compute the phase-portraits of complex dynamics as well as the center, stable and unstable manifolds originating at the partially hyperbolic equilibria, and the Lagrangian Coherent Structures of aperiodic flows. While the definition of most chaos indicators is clearly inspired by the Characteristic Lyapunov Exponent theory, their use is oriented to extract all the information which is contained in the solutions of the variational equations in short time intervals. In this lecture I will show through examples why the computation of short time chaos indicators is particularly powerful for those systems whose solutions may have an asymptotic behaviour very different from the short-term one, as it can be the case of sequences of close encounters in gravitational systems and the advection of particles in aperiodic flows. The main case study will be the computation of transit orbits in the restricted three-body problem.

Introduction to KAM theory and applications

U. Locatelli

KL

University of Rome Tor Vergata, Rome, Italy

The very first application of KAM theory to Celestial Mechanics was made soon after its birth, nearly 60 years ago [Leontovich, Soviet Math. Dokl. (1962)]. This is not by chance, Kolmogorov himself [Dokl. Akad. Nauk SSSR (1954)] emphasized that Celestial Mechanics was a natural domain for applying his theorem.
Reviewing the strict relation between KAM theory and Celestial Mechanics is the main purpose of our lecture. Actually, modern applications often combine different normal forms. Here, KAM theory is first introduced by discussing in a unified way three algorithms, which construct the usual (maximal) invariant tori, the (lower-dimensional) elliptic tori and also the secular approximation of order two in the masses, that is deeply studied for what concerns the planetary dynamics. This is made by adapting to our purposes the approach developed by Pöschel [Math. Z. (1989)]. The second part of the lecture deals with the applications to exoplanetary systems, by focusing on a sort of KAM reverse method that is designed so to estimate the (unknown) values of the mutual inclinations. In particular, some results previously obtained [Volpi et al., CMDA (2018)] are discussed and the method to improve them is sketched. Actually, such a new approach is based on a careful combination of all the constructive algorithms described in the first part of the lecture, that is devoted to the introduction of KAM theory.

This work is based on a research project made in collaboration with C. Caracciolo, M. Sansottera and M. Volpi.

The *n*-body problem

G. Pinzari

KL

University of Padova, Padua, Italy

As a breakthrough application of Kolmogorov's theorem on the conservation of invariant torus, V.I.Arnold, in the 60s, formulated a theorem on the stability of planetary motions for infinite times [11][2]. The proof given in [6] relies on a system of Poincaré-like system of canonical coordinates well suited to the $SO(3)$ invariance of the system. Since then, other systems of coordinates have been discovered, leading to different results; e.g., [18]. In these lectures, we shall highlight the importance of canonical coordinates in the perturbative approach to the *n*-body problem.

References

[1] V.I. Arnold. Sibirsk. Mat. Ž., 4:471–474 (1963).
[2] V.I. Arnold. Russian Math. Surveys, 18(6):85–191 (1963).
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Contributed Talks

Constraining the mass of the graviton with the planetary ephemeris INPOP

L. Bernus[1], **O. Minazzoli**[3,4], **A. Fienga**[1,2], **M. Gastineau**[1], **J. Laskar**[1], **P. Deram**[2]

CT

[1] IMCCE, Paris, France
[2] Côte d'Azur Observatory, Valbonne, France
[3] Centre Scientifique de Monaco, Monaco
[4] Côte d'Azur Observatory, Nice, France

We use the planetary ephemeris INPOP17b to constrain the existence of a Yukawa suppression to the Newtonian potential, generically associated with the graviton's mass. We also give a n interpretation of this result for a specific case of fifth force framework. We find that the residuals for the Cassini spacecraft significantly (90% C.L.) degrade for Compton wavelengths of the graviton smaller than 1.83×10^{13} km, which correspond to a graviton mass bigger than 6.76×10^{-23} eV/c^2. This limit is comparable in magnitude to the one obtained by the LIGO-Virgo Collaboration in the radiative regime [1]. We also use this specific example to defend that constraints on alternative theories of gravity obtained from post fit residuals may be generically overestimated. This study has been published in PRL [2].

References

[1] The LIGO Scientific Collaboration and the Virgo Collaboration. Tests of General Relativity with the Binary Black Hole Signals from the LIGO-Virgo Catalog GWTC-1 (2019). `http://arxiv.org/abs/1903.04467`
[2] L. Bernus, O. Minazzoli, A. Fienga, M. Gastineau, J. Laskar, P. Deram: *Constraining the Mass of the Graviton with the Planetary Ephemeris INPOP*. Phys. Rev. Lett., 123:161103 (2019). `https://doi.org/10.1103/PhysRevLett.123.161103`

Analysis of parameters space for Intermediaries of the roto-orbital problem

J. Cardoso dos Santos

CT

Israel Institute of Technology, Haifa, Israel

This work presents an updated analysis of two Hamiltonians that produce integrable models recently proposed to study the roto-orbital motion of an axisymmetric rigid body in motion under the influence of a central gravitational field. The dynamics assumed in both models is the motion of an axisymmetric rigid body orbiting another massive spherical one. Using canonical variables associated to the total angular momentum and based on the concept of intermediary, both models are described in a Hamiltonian formalism of the disturbed Keplerian-Eulerian motion. Analysis of parameters introduced to test precision and ranges of validity for different applications are presented. In the present applications, special focus given to the case binary asteroid type dynamics. The parameters space analysis is made by comparing the two proposed intermediaries with respect to the original non-analytically integrable model and with respect to each other. The conclusion is that both models present significant precision in the regions of the parameters space where they were proposed to be valid. This work is based on [1], [2] and [3].

References

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[2] J. Cardoso dos Santos, JPCS, 2019 (Accepted).
[3] J. Cardoso dos Santos, S. Ferrer, D.J. Scheeres, CM&DA, 131:26 (2019).

COMPASS project for space debris modelling and mitigation

C. Colombo

CT

Politecnico di Milano, Milan, Italy.

Space benefits humanity by making many of our activities on Earth possible: telecommunications, weather forecasting, geolocation, maritime and air traffic management and remote sensing to monitor the health of our planet. At the same time, scientific missions increase knowledge of our solar system and allow the development of new technologies, science and space exploration. Humans routinely venture beyond Earth and send spacecraft into orbit to provide ground-based services and explore other planets. This extraordinary ability, however, calls for a great responsibility: our space activity risks contaminating the space that surrounds the Earth.
The COMPASS project proposes to leverage the dynamics of natural orbit perturbations and the tools of dynamical system theory and uncertainty propagation to model the evolution of space debris and their related uncertainties and to reduce delta-v cost associated to the design of disposal manoeuvres.In this talk we will present the theoretical advancement of the COMPASS project in the space debris modelling and remediation. The use of a continuum approach is used to describe the evolution of the spatial density of space debris fragments in orbit as function of time,or to compute the dispersion of uncertain initial conditions at re-entry. For the design of disposal manoeuvres, a phase space representation is used to model the effect of orbit perturbations, such as luni-solar effect,and the effect of artificial changes to the orbit,such as delta-v manoeuvre and low-thrust propulsion.The phase space representation is used to optimise the artificial manoeuvre for achieving re-entry.The proposed idea of optimal navigation through orbit perturbations can address various major engineering challenges in space situation awareness, for application to space debris evolution and mitigation, re-entry prediction.

This project has received funding from the European Research Council(ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 679086 -COMPASS).

Manifold structure and chaotic ballet of navigation satellites

J. Daquin[1], **I. Gkolias**[2], **C. Efthymiopoulos**[1]

CT

[1] University of Padova, Padua, Italy
[2] Politecnico di Milano, Milan, Italy.

In this talk we reveal a manifold structure in a 3 degrees-of-freedom Hamiltonian system physically representative of the dynamics of navigation satellites. In particular, long-time properties of the system are explained in terms of the possible normally hyperbolic manifold structure and the associated stable and unstable manifolds regulating transport properties. Besides capturing the region of hyperbolicity of quasi-circular orbits near a Lidov-Kozai resonance, the resulting chaotic escape we describe could inspire the design of end-of-life disposals strategies using manifold dynamics.

J.D. is funded by the ERC project 677793 "Stable and Chaotic Motions in the Planetary Problem".

Numerical evidence of symbolic dynamics in a three-body problem

S. Di Ruzza, J. Daquin, G. Pinzari

CT

University of Padova, Padua, Italy

We consider the secular Hamiltonian of a planar three-body problem with two equal masses, in jacobian coordinates. After finding a periodic orbit, we describe its dynamics through the Poincaré map, reducing the problem to a 2 degree-of-freedom discrete system. We analyse the phase space and classify elliptic and hyperbolic structures. Thanks to the local knowledges of the manifolds structures, we are able to numerically prove the existence of symbolic dynamics. We construct an explicit horseshoe around one heteroclinic intersection using ad-hoc coverings patches. The analysis is numerical and uses correctly aligned windows, as described in a recent paper by A. Gierzkiewicz and P. Zgliczyński, combined with a recent set of canonical coordinates for the three-body problem.

The work is funded by ERC project 677793 “Stable and Chaotic Motions in the Planetary Problem”.

Invariant Manifolds near L_1 and L_2 in the Planar Elliptic Restricted Three-Body Problem

G. Duarte[1,2], **À. Jorba**[1,2]

CT

[1] Universitat de Barcelona, Barcelona, Spain
[2] Barcelona Graduate School of Mathematics, BGSMath, Spain

In this work we investigate the connections between the stable and unstable manifolds of periodic orbits around the points L_1 and L_2 of the Restricted Three-Body Problem (RTBP).
In the Planar Circular RTBP (PCRTBP) we can see, based on [1] (and its references), that there is a connection of these manifolds on the phase space of this problem, which means that there is a mechanism of an orbit that goes outside the orbit of one primary (for instance, that describes an orbit close to an ellipse with greater semi-major axis), to go through it and start to d escribe an orbit inside of it (close to an ellipse with smaller semi-major axis) and/or vice-versa; in other words, this mechanism is a bridge connecting orbits outside with orbits inside.
In addition, the Elliptic problem (PERTBP) is also studied. In this case (where the eccentricity of Jupiter's orbit is considered) the periodic orbits around L_1 and L_2 are now tori. Using the methodology described in [2], we analyse these tori and their stable and unstable manifolds in order to: (i) compare the accuracy of the computations to describe these orbits going from outside to inside and vice-versa, and (ii) to check the improvement in describing this type of movement using the PERTBP.

We acknowledge the Spanish Ministry of Economy and Competitiveness, through the María de Maeztu Programme for Units of Excellence in R&D (MDM-2014-0445).

References

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Extremely long-term asteroid propagation

O. Fuentes-Munoz, D. J. Scheeres CT

University of Colorado Boulder, Boulder, Colorado.

Asteroid orbits evolve chaotically over millions of years driven by planetary encounters. Long-term simulations are possible by treating the encounters analytically. In this paper we propose a methodology to predict the next encounter date as well as the effect of the flyby on the asteroid orbit elements. Far encounters occur with high frequency but break the assumptions of analytical solutions. In order to account for such events Lagrange Planetary Equations are integrated around the closest approach date using modified Delaunay elements. The theory assumptions are validated by comparison of the simulations with numerical integrations at different timescales.

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Andrade rheology in time-domain. Application to Enceladus' dissipation of energy due to forced libration

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CT

The main purpose of this work is to present a time-domain implementation of the Andrade rheology, instead of the traditional expansion in terms of a Fourier series of the tidal potential. This approach can be used in any fully three dimensional numerical simulation of the dynamics of a system of many deformable bodies. In particular, it allows large eccentricities, large mutual inclinations, and it is not limited to quasi-periodic perturbations. It can take into account an extended class of perturbations, such as chaotic motions, transient events, and resonant librations.

The results are presented by means of a concrete application: the analysis of the libration of Enceladus. This is done by means of both analytic formulas in the frequency domain and direct numerical simulations. We do not a priori assume that Enceladus has a triaxial shape, the eventual triaxiality is a consequence of the satellite motion and its rheology. As a result we obtain an analytic formula for the amplitude of libration that incorporates a new correction due to the rheology.

Our results provide an estimation of the amplitude of libration of the core of Enceladus as 0.6% of that of the shell. They also reproduce the observed 10 GW of tidal heat generated by Enceladus with a value of 0.17×10^{14}Pa·s for the global effective viscosity under both Maxwell and Andrade rheology.

State-dependent perturbation of planar ODEs

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We consider an analytic 2 dimensional ordinary differential equation $\frac{dx}{dt}(t) = X_0(x(t))$, which admits a limit cycle. We consider the singular perturbation of adding a state dependent delay.

$$\frac{dx}{dt}(t) = X(x(t), \varepsilon x(t - r(x(t)))), \qquad 0 \leq \varepsilon \ll 1. \tag{1}$$

We show that for small enough ε, there exist a limit cycle and a two dimensional family of solutions to the perturbed state dependent delay equation (1) which resemble the solutions of the unperturbed ODE. More precisely, we find a parameterization of the limit cycle and its stable manifold for the perturbed equation.

An analytical insight into the end-of-life disposal of close-Earth satellites using a solar sail

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CT

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Earth satellite orbits, akin to other natural resources available to humankind, should be used in a sustainable way. The continuously increasing number of resident space objects yields efficient end-of-life disposal strategies. Depending on the satellite's altitude above the Earth, natural perturbations of different type can be exploited to achieve this goal. For satellites in Low Earth Orbits (LEO), the drag due to Earth's atmosphere dictates the orbital lifetimes. However, this effect decays exponentially and satellites at a few thousand kilometres altitude could remain at their orbits for centuries. Remarkably, higher LEO and Medium Earth Orbits have also been proposed to host large satellite constellations. Nevertheless, the disposal process can be enhanced via an area augmenting device. The coupled effect of Earth's oblateness and solar radiation pressure (SRP) creates favourable conditions for fast eccentricity growth and eventual atmospheric re-entry. A harmonic decomposition of the averaged SRP force allows to identify the critical arguments that dominate locally the dynamics. Resonant Hamiltonian constructions provide a significant analytical insight in the neighbourhood of each resonance. De-orbiting conditions as well as the associated time-scales are retrieved by analysing the phase space of the resulting 1 Degree-of-Freedom models. This information allows to engineer the control parameters, like the satellite area-to-mass ratio, as well as to target favourable conditions in order to comply with international regulations about the post-mission orbital lifetimes. The validity of the analytical results is investigated under more complete force models.

Part of the research leading to these results has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme as part of project COMPASS (Grant agreement No 679086).

Arnold diffusion for a Hamiltonian with $3 + 1/2$ degrees of freedom

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CT

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In the present work we study the geometrical mechanism of diffusion in an *a priori* unstable Hamiltonian system with 3 + 1/2 degrees of freedom

$$H_\varepsilon(p, q, I_1, I_2, \varphi_1, \varphi_2, s) = \pm\left(\frac{p^2}{2} + \cos q - 1\right) + h(I_1, I_2) + \varepsilon f(q)\, g(\varphi_1, \varphi_2, s), \quad (1)$$

where $f(q) = \cos q$, $h(I_1, I_2) = \Omega_1 \frac{I_1^2}{2} + \Omega_2 \frac{I_2^2}{2}$, $(\varphi_1, \varphi_2) \in \mathbb{T}^2$ and

$$g(\varphi_1, \varphi_2, s) = a_1 \cos\varphi_1 + a_2 \cos\varphi_2 + a_3 \cos s. \quad (2)$$

Combining iterates of the *inner* and the *outer* dynamics associated to a 5D-*Normally Hyperbolic Invariant Manifold* (NHIM) to build a diffusing *pseudo-orbit* and applying Shadowing results we prove the existence of a diffusing orbit of the system. More precisely, we are able to prove the following theorem on global instability

Theorem 1. *Consider the Hamiltonian* (1)+(2). *Assume* $a_1, a_2, a_3 \neq 0$ *and* $|a_1/a_3| + |a_2/a_3| < 0.625$. *Then, for every* $\delta < 1$ *and* $R > 0$ *there exists* $\varepsilon_0 > 0$ *such that for every* $0 < |\varepsilon| < \varepsilon_0$, *given* $|I_\pm| \leq R$, *there exists an orbit* $\tilde{x}(t)$ *and* $T > 0$, *such that*

$$|I(\tilde{x}(0)) - I_-| \leq \delta \quad \text{and} \quad |I(\tilde{x}(T)) - I_+| \leq \delta.$$

Long-term destabilisation of Mercury orbit: the dynamical mechanism

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Numerical integrations of averaged (Laskar 1994, 2008) and complete equations of motion (Laskar & Gastineau 2009) have shown the possibility of destabilisation of Mercury orbit within the next 5 billion years, with a probability of about 1%. Such an instability have been demonstrated to be due to an entrance into the linear secular resonance $g_1 - g_5$, between the precession frequencies of Mercury and Jupiter perihelion longitudes (Laskar 2008, Batygin & Laughlin 2008, Boué et al. 2012).
The timescale for Mercury destabilisation is much longer than the Lyapunov time of the inner solar system, which sets the chaotic decorrelation timescale of initially-close trajectories and is of about 5 million years (Laskar 1989). To date, the only work which addresses the reason of such different timescales (Batygin et al. 2015) focuses on the degrees of freedom related to Mercury orbit, freezing all the other planets on quasi-periodic orbits. The typical Mercury destabilisation timescale is predicted to be $10^8 - 10^9$ yr, which is 10 to 100 smaller than what should be expected from the statistics of numerical integrations.
Our work is based on a new model of secular dynamics for the inner solar system (Mogavero & Laskar, in prep.), with the giant planets following predetermined quasi-periodic orbits, and the inner ones moving in the resulting time-dependent external potential. A systematic study of the Hamiltonian terms driving the chaotic diffusion towards the $g_1 - g_5$ resonance has been performed through the computer algebra system TRIP (Gastineau & Laskar 2011). We present our current results.

An application of the Flyby map to Europa Clipper mission

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Europa Clipper is a NASA scientific mission to study Europa and answer the question about its potential sustainability of existing life. The presence of liquid water, heat source and different chemical elements place Europa in the top priority list for NASA planetary exploration program. The Europa Clipper's trajectory is a multi-moon orbiter mission accounting over 45 Europa, 5 Ganimede and 9 Callisto flybys answering to several design challenges: high resolution distributed observations of the surface of Europa under different luminosity conditions, high datalink and low radiation dose exposure, which constitutes a critical point considering that Europa resides deep inside Jupiter magnetosphere.

The trajectory exploits the gravitational interaction of the Galilean moons with Jupiter field but its design is still based on a patched conics modelling of such effects. Nevertheless, planetary moon systems, such as Jupiter one, exhibits strong perturbations that induce large errors in position and velocity on the two-body trajectory. Such discrepancies result in large delta-v and therefore in poor convergence when the orbit is refined in the full-model, which considers the dynamics of the six-body problem represented by Jupiter and the Galilean moon system together with the perturbation induced by the Sun and Jupiter J2 effect.

The idea behind this paper is to apply the Flyby map to improve the design of Europa Clipper tour. The method numerically maps the non-linear effect of the flyby in the phase space of the Keplerian elements. Differently from the patched conics which model this effect with analytical formula, the Flyby map numerically propagates the cartesian state in the three-body dynamics between two Poincaré sections and studies the variation of semi-major axis and inclination induced by the osculating longitude and the argument of the periapsis that modify the condition at the close approach.

More than two thirds of Clipper flybys about Europa consists in resonant flyby that foresees an increment and subsequent reduction of the inclination about Jupiter to achieve a specific distribution of the close approaches over different quadrants of the icy-moon surface.

Differently from what patched conics theory suggests, the Flyby map allows to distinguish the effect of prograde and retrograde flybys, indicates the existence of a natural sequence of resonant flybys that if followed lead to low delta-vs and offer initial conditions that are more prone to converge in the full-body dynamics. The paper describes in detail the generation of initial conditions for resonant flybys derived from the interpolated result of the Flyby map, their optimisation in the circular restricted three- and full-body dynamics and answer whether it exists a manifold in the Europa-Jupiter systems to which belongs the sequence of resonant flybys and how it modifies under the effect of the perturbations considered.

The specific property of motion of resonant asteroids with very slow Yarkovsky drift speeds

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The subject of our research is interaction between the mean motion resonance (MMR) and the Yarkovsky effect. This interaction occurs when an asteroid due to the changes of its orbital semi-major axis (caused by the Yarkovsky effect) reach the resonance. The resonance induces a periodic oscillations in the asteroid's semi-major axis around its center. The Yarkovsk effect exactly causes the permanent (secular) evolution of the orbital semi-major axis. As a result of their interaction, the mean semi-major axis drift speed is modified with respect to the one caused solely by Yarkovsky effect. We examined the specific characteristic of motion of asteroids with 9 very slow Yarkovsky drift speeds (da/dt) across the 11 two-body mean-motion resonances with Jupiter whose strengths cover a wide range. We observed only asteroids that crossed the MMR. This investigation was carried out with numerical integrations performed in the public domain integrator ORBIT9. We noticed that the test asteroids moved extremely fast across MMRs with very small Yarkovsky drift speeds (order of magnitude 105 au/Myr or less). This result may indicate that below a certain boundary value of da/dt, asteroids typically go quickly across MMRs. From the obtained results, we concluded that the boundary value of the Yarkovsky drift speed is at 7 105 au/Myr. Possible explanation about results of our research is that MMRs have more powerfully influence than very slow Yarkovsky drift speeds on motion of resonant asteroids, especially the stronger ones.

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Origin of Chaos and Resonant Structure in the Inner Solar System

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Despite its discovery dating back to thirty years ago, there is still no full consensus in scientific literature about the origin of chaos in the planetary orbits of the inner solar system.

Laskar (1990) measured the libration period of the resonant argument $\theta_{2:1} = 2(\varpi_4^* - \varpi_3^*) - (\Omega_4^* - \Omega_3^*)$, involving Mars and Earth perihelion and node longitudes, to be about 4.6 million years. He thus proposed its transitions between libration and circulation as an explanation for the observed maximal Lyapunov exponent (MLE) of about 5 Myr. In a following analysis, he gave evidence of a large chaotic zone spanning from the resonance $\theta_{2:1}$ to $\theta_{1:1} = (\varpi_4^* - \varpi_3^*) - (\Omega_4^* - \Omega_3^*)$ (Laskar 1992). Nevertheless, when the integration of the full equations of motion confirmed the MLE of the averaged ones (Sussman & Wisdom 1992), the chaotic nature of the libration-circulation transitions of $\theta_{2:1}$ was questioned. In the meantime, Laskar's explanation has been supported by direct integrations (Laskar 2004) and the alternating librations of $\theta_{2:1}$ and $\theta_{1:1}$ confirmed by geological data (Ma et al. 2017, Zeebe & Lourens 2019).

We developed a new model of secular dynamics for the inner solar system (Mogavero & Laskar, in prep.), with the giant planets following predetermined quasi-periodic orbits, while the inner ones move in the resulting time-dependent external potential. A systematic study of the corresponding Hamiltonian has been performed exploiting the computer algebra system TRIP (Gastineau & Laskar 2011). The Hamiltonian terms generating the 5-Myr Lyapunov time have been isolated, establishing beyond any doubt the precise dynamical origin of chaos. We also derived a systematic list of the secular resonances coming into play, which reveals an intricate dynamics both among the inner planets, and between the inner and outer systems.

Study of large Lyapunov orbits and temporary captures via the Levi-Civita Hamiltonian normalization

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The dynamics near the collinear Lagrange equilibria L_1 and L_2 of the Circular Restricted Three-body Problem has gained attention in the last decades due to its relevance in some topics such as the temporary captures of comets and asteroids, and the design of trajectories for space missions. In this paper we investigate the temporary captures using the tube manifolds of the horizontal Lyapunov orbits originating at L_1 and L_2 of the CR3BP, at values of the energy which have not been considered so far. After showing that the radius of convergence of any Hamiltonian normalization at L_1 or L_2 computed with the Cartesian variables is limited in amplitude by $|1-\mu-x_{L_1}|$ (μ denoting the reduced mass of the problem), we investigate if regularizations allow us to overcome this limit. In particular, we consider the Hamiltonian describing the planar three-body problem in the Levi-Civita regularization and we compute its normalization for the Sun-Jupiter reduced mass for an interval of values for the energy which overcomes the limit of Cartesian normalizations. As a result, for the largest Lyapunov orbits that we consider, we notice a transition of topology in the structure of the tubes manifolds emanating from the Lyapunov orbit, which in particular can contain orbit which collide with the secondary body before perfoming one full circulation around it. We discuss the relevance of this transition for the possibility of collisions for the orbits which are temporary captured by the secondary body.

Utilizing Chaos for Debris Mitigation

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CT

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In the past five years there has been a resurgence of dynamical studies of medium Earth orbit. In the 1990's GPS regions were studied for their chaotic behavior. However, with the recent launches of Galileo, the European project RedShift has shown renewed interest in studying the long term dynamics of objects in this region[1]. Medium Earth orbit is subject to chaotic effects due to third body perturbations. Interaction between luni-solar resonances causes a rapid increase in eccentricity of these orbits. Resonance structure has been described by Rosengren et al. including possible areas for chaos in the region due to crossing resonances [1].

In this talk, I plan to discuss these dynamical structures. In particular, I will be discussing my contribution to this work. This includes the robustness of targeting these regions for atmospheric reentry which involves understanding the sensitivity of these regions [2]. I have also studied the long term dynamics of potential debris in this region such as mylar shedding which is done through studying high area-to-mass ratio, HAMR, objects.

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The onset of instability in resonant chains

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Planets smaller than Saturn, such as Super-Earths and Mini-Neptunes, undergo so-called type-I migration in the proto-planetary disc in which they form. When multiple planets are orbiting the same star, the inner one stops migrating at the inner edge of the disc and a favoured outcome of the slow, convergent orbital transport of the planets is the formation of a chain of mean motion resonances. When the disc is dissipated away and the damping forces applied by the disc are removed, the resulting system may be dynamically stable or unstable. Observations show that only a small fraction of these resonant chains should be stable, and these instabilities are indeed found in N-body simulations. In particular, previous numerical simulations showed that the maximum planetary mass for a resonant to remain stable decreases with increasing number of planets and increasing compactness of the system, but the origin of the instability was not understood. We focus on the case of three equal-mass planets in the 3:2 – 3:2 mean motion resonant chain and we study which dynamical phenomena lead to a loss of stability if the planetary mass is larger than some threshold. We identify a set of secondary resonances between the libration frequencies in the mean motion resonance and a sub-frequency of the synodic period $2\pi(1/P_1 - 1/P_2)$. These secondary resonances excite the amplitudes of libration of the mean motion resonances. We generalise this mechanism to different chains of mean motion resonances involving a larger number N of planets and/or a higher index k of the $k : k-1$ mean motion resonances, obtaining an analytical scaling for the maximal planetary mass that guarantees stability, as a function of N and k. The results are in qualitative agreement with the numerical experiments mentioned above, thus giving a theoretical understanding of the instabilities observed in the simulations, as well as a better grasp on the fingerprint of the planetary dynamical history that is contained in the current exoplanets sample.

Modeling of three-dimensional coupling of spin,

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In this talk, we will discuss a system of ordinary differential equations for the motion of linear viscoelastic bodies interacting under gravity. The equations are fully three dimensional and allow for the integration of the spin, the orbit, and the deformation of each body. Using such a formulation, we can present good models for the tidal forces that take into account the possibly different rheology of each body. The equations are obtained within a finite dimension Lagrangian framework with dissipation function. The main contribution is a procedure to associate to each spring-dashpot model, which defines the rheology of a body, a potential and a dissipation function for the body deformation variables. The theory applied to the Earth enables a comparison between model and observation of the quantities: norm of the Love numbers, rate of tidal energy dissipation, Chandler period, and Earth-Moon distance increase. The theory is also used to study the libration of the Moon and other satellites of the Solar System.

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Lectures and Talks will be held at the **Aula Chisini** of the Department of Mathematics "Federigo Enriques" of the University of Milan. It is situated on the ground floor of the building (entering from the main entrance, just go straight and cross the courtyard).

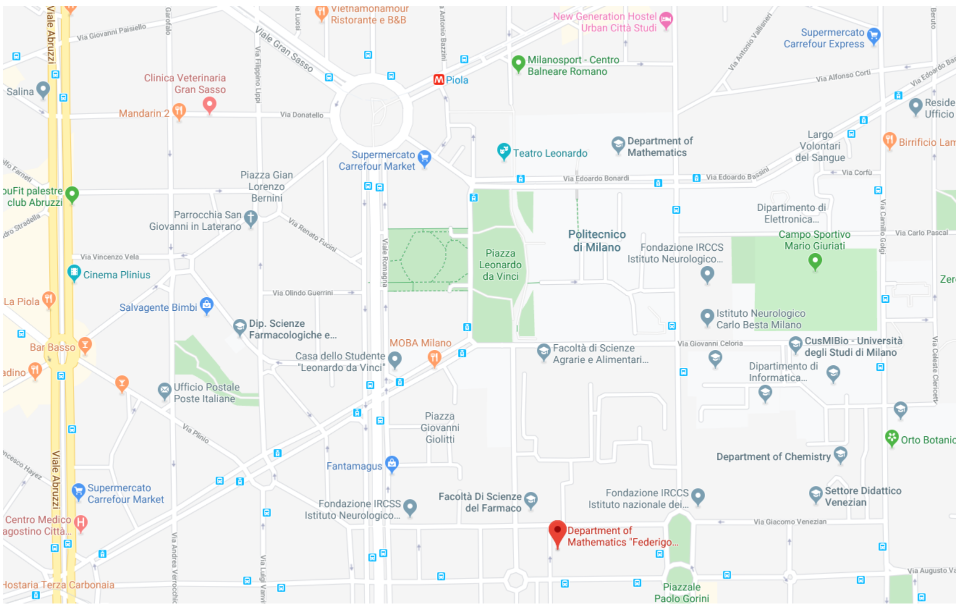
Coffee breaks will be offered in the **Sala Rappresentanza**.

Wi-Fi will be available during the conference via eduroam network.

How to get to the Department of Mathematics "Federigo Enriques"?

The Department of Mathematics "Federigo Enriques" is located in via Cesare Saldini 50, 20133, Milan, Italy, and can be reached by:

- **Metro: Green Line** (**Line 2** of the Metro Underground) station **Piola**.
- **Bus:** line **61**, stop "Via Botticelli/Via Saldini"



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These are some possible locations where you can have lunch nearby the school location.

1. Union — via Moretto da Brescia 36
2. Mida — via Sandro Botticelli 22
3. Bar Avanti C'è Posto — via Sandro Botticelli, 2
4. Strambio 6 — via Gaetano Strambio 6
5. Le Café de Flore — via Gaetano Strambio 3
6. Il focacciaro — via Gaetano Strambio 6 (mainly take away)
7. Kilomangiaro — viale Romagna 58 (only take away)
8. Lissana — piazzale Gorini 9 (only take away)
9. Focacceria Città Studi — piazzale Gorini 11 (only take away)
10. Trattoria pizzeria la pergamena — via Sandro Botticelli 13 (only take away)

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