Long-term destabilisation of Mercury orbit

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Chaotic solar system

- Solar system is chaotic (Laskar, 1989, 1990)

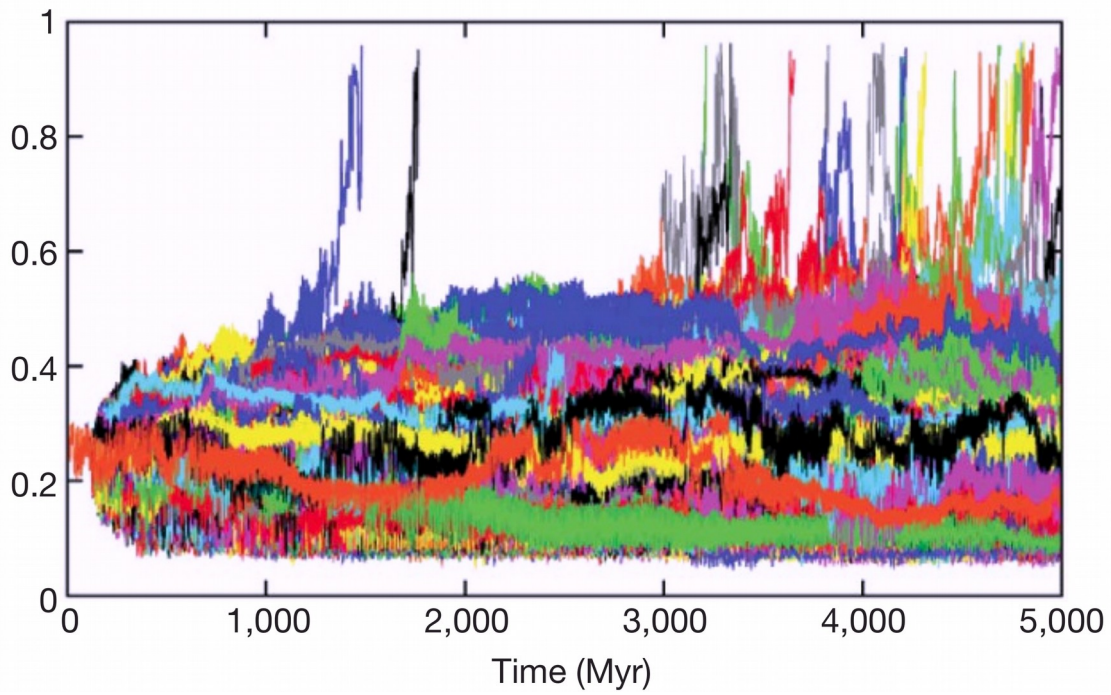
$$d = d_0 \, \mathrm{e}^{T/t_{lyapunov}}, t_{lyapunov} \approx 5 \, Myrs$$

- It is impossible to trace the Solar System solution for longer than ~ 100 Myrs from initial condition, now matter how precise the initial condition
- Question: In the next 5 Gyrs, will the Solar System still be in a vigorous, yet bounded chaos. Or is a catastrophic event possible?

Ejection of Mercury

- From 2500 solutions directly integrated from slightly different initial condition, ~1 % of ejection of Mercury from Solar System

Eccentricity
Of Mercury

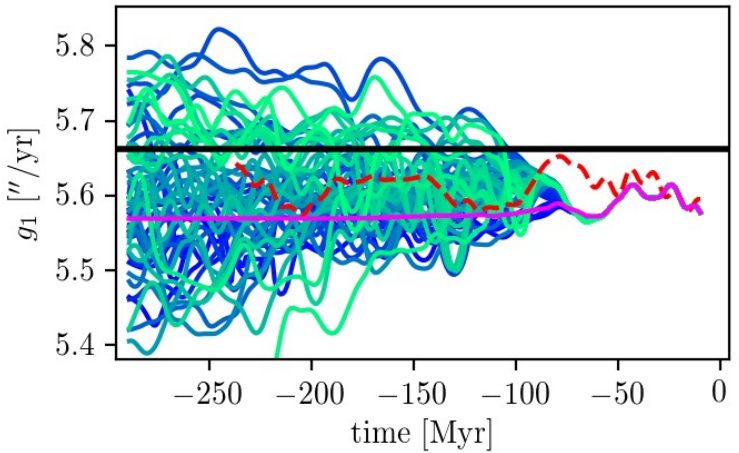


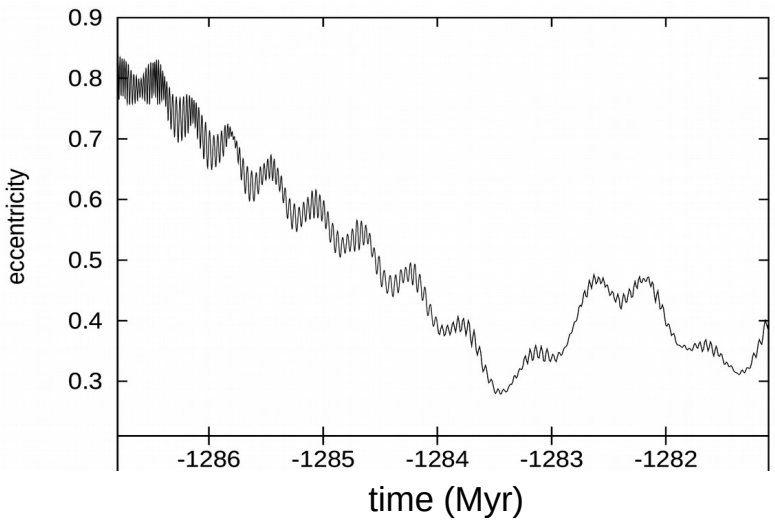
(Laskar and Gastineau 2009)

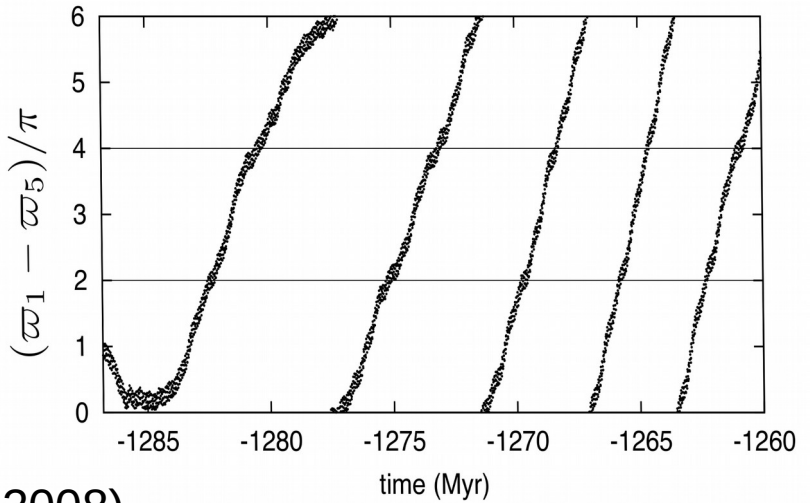
Resonance of apsidal precession

- Currently, secular perihelion precession frequency of Mercury: g_1 = 5.59 [‘’/year],
  of Jupiter: g_5 = 4.54 [‘’/year]

- Violent growth of Mercury’s eccentricity when $g_1 = g_5$







(Laskar 2008)

A simple model of chaotic Mercury

Secular Hamiltonian of planetary motions:

$$H(\boldsymbol{I},\boldsymbol{\Phi}) = H_{LL}(\boldsymbol{I}) + \sum_{\boldsymbol{k}\in\mathbb{Z}^{16}} A^{k}(\boldsymbol{I})\cos(\boldsymbol{k}\cdot\boldsymbol{\Phi})$$

- H_{LL} is the integrable Laplace-Lagrange Hamiltonian
- ($\boldsymbol{I}$, $\boldsymbol{\Phi}$) are the planetary action-angle variables
- $\boldsymbol{k}$ is a vector of integer corresponding to a harmonics
- A^{k} are functions of actions variables.

Simple model of Mercury (Batygin et al. 2015, Boué et al. 2012, Lithwick et Wu 2011)

- Massless Mercury is perturbed by the others planets whose orbits are quasi-periodic functions.
- All harmonics are eliminated except: Harmonics associate with perihelion of Mercury, Venus and Jupiter ($g_1 - g_5$, $g_1 - g_2$) and longitude of ascending node of Mercury and Venus ($s_1 - s_2$).
- Hamiltonian is up to order 4 in eccentricity and inclination

A simple model of chaotic Mercury

Chaotic Hamiltonian with 2 degrees of freedom

$$H_0 = (g_5 - g_{1,0})\, I + A_1 I^2 + A_2 \sqrt{I} \cos(\phi) + (s_2 - s_{1,0})\, J + B_1 J^2 + B_2 \sqrt{J} \cos(\psi) + C\, I\, J$$

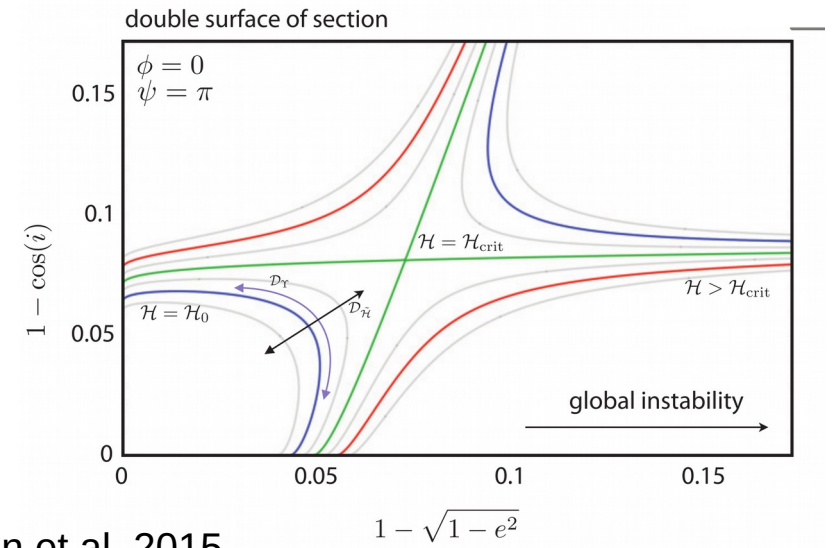
Angles ψ , ϕ are conjugate to actions

$$I = 1 - \sqrt{1-e^2}, J = \sqrt{1-e^2}(1-\cos i)$$

=> Chaotic with $t_{lyapnunov}$ ~ 1 Myr but bounded because of constant energy

H_{crit} corresponds to null frequencies of ψ , ϕ , *i.e* $g_1 = g_5$ *and* $s_1 = s_2$

Level curve of H_0(*I, J*)



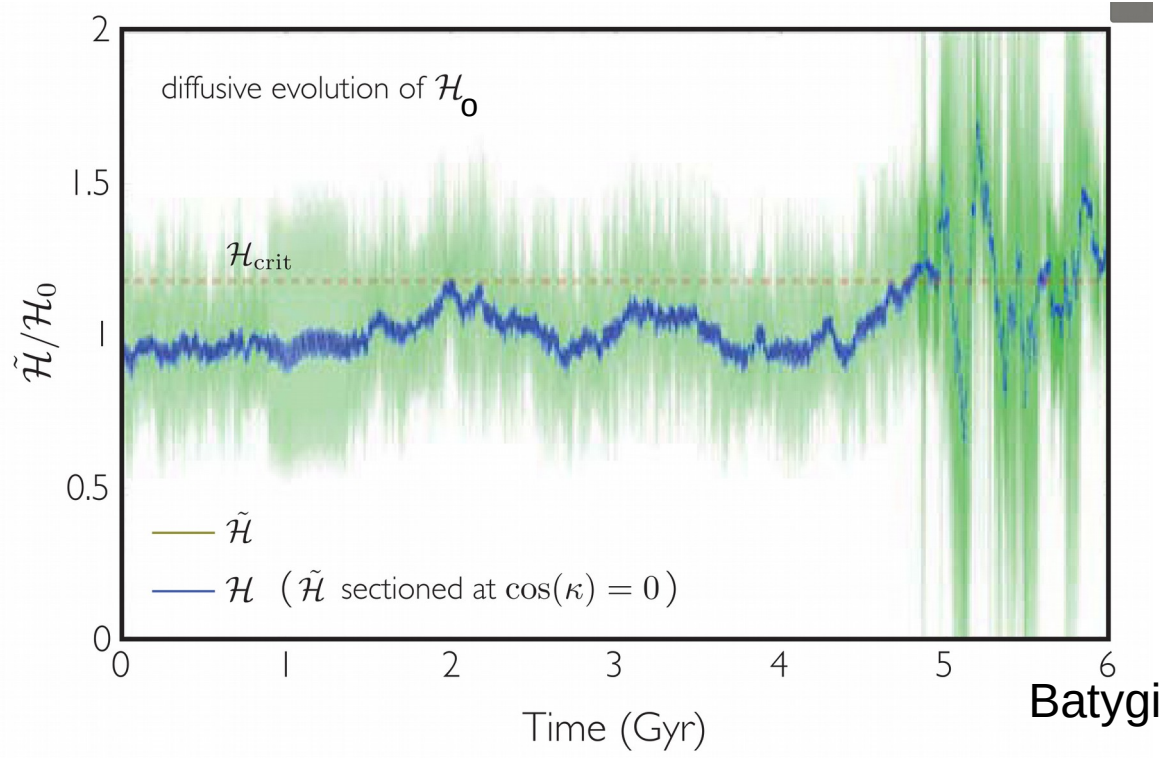
Batygin et al. 2015

A simple model of chaotic Mercury

Perturbed chaotic Hamiltonian with 2.5 degrees of freedom:

$$H = H_0(I, J, \phi, \psi) + A_T \sqrt{I} \cos(\phi + (g_2 - g_5)t + \beta)$$

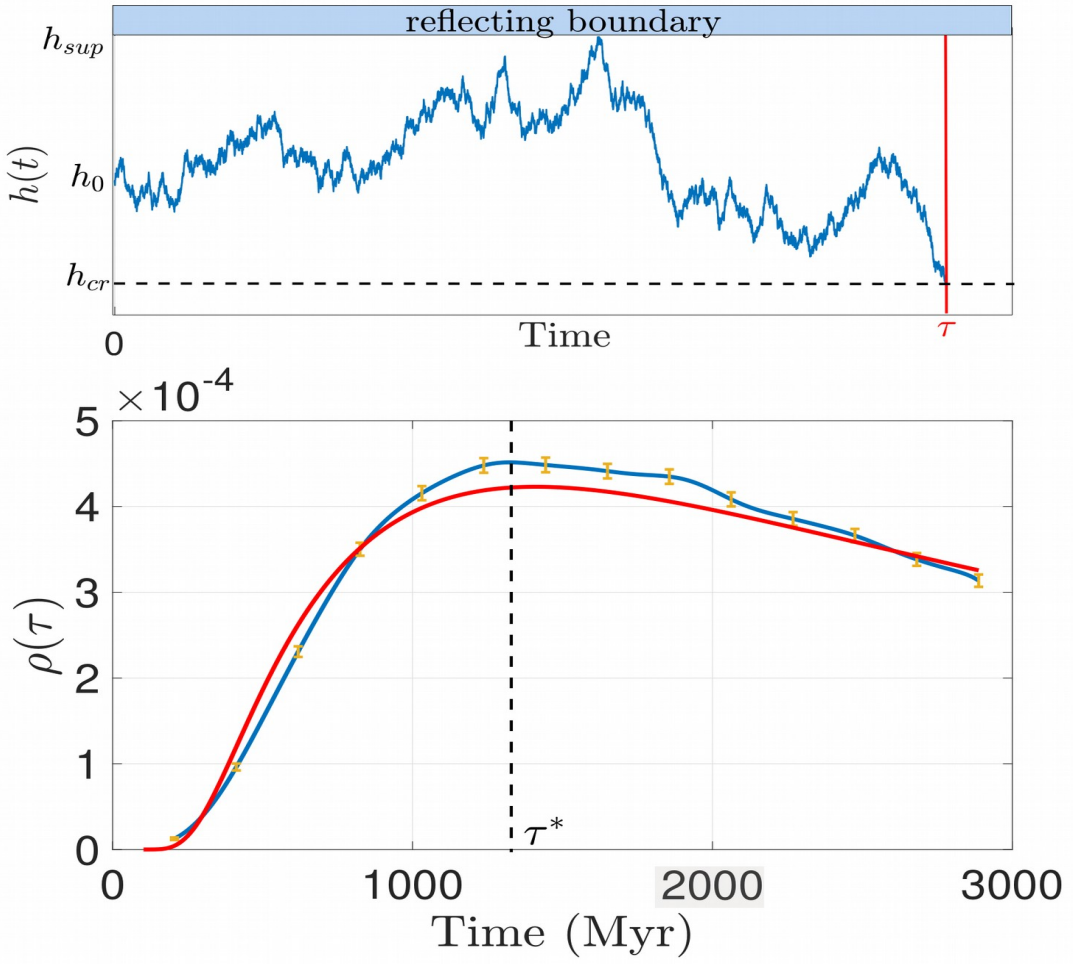
The system is quickly destabilized when H_0 reaches H_{crit}.



Batygin et al. 2015

A simple model of chaotic Mercury

- A simple Brownian motion can describe well the statistics of this simple model
- $T_{destabilasation}$ = 1 Gyr >> $T_{lyapunov}$ = 1 Myr,
- But at least 10-100 times smaller than the actual system (1% for 5 Gyr)



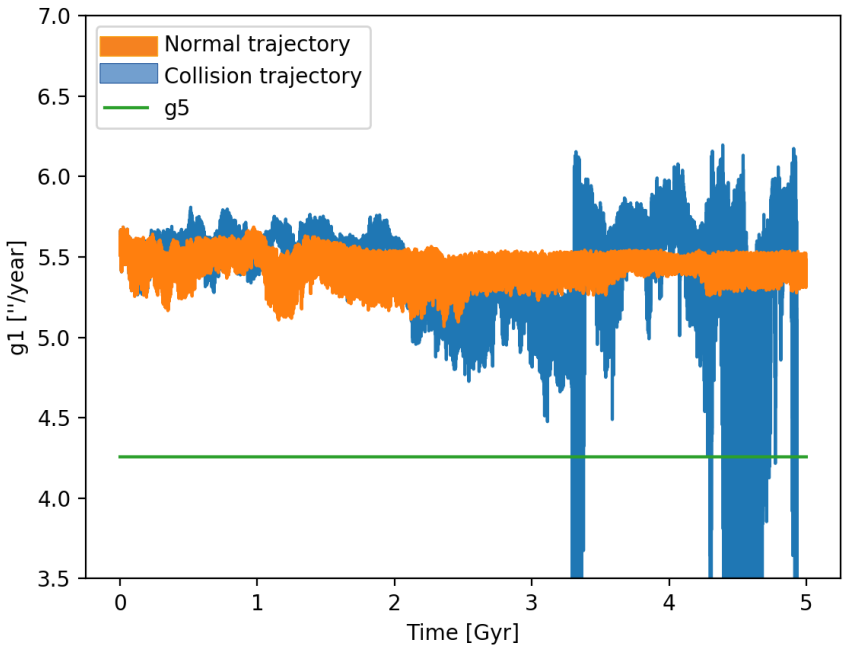
Bouchet and Woillez (2019)

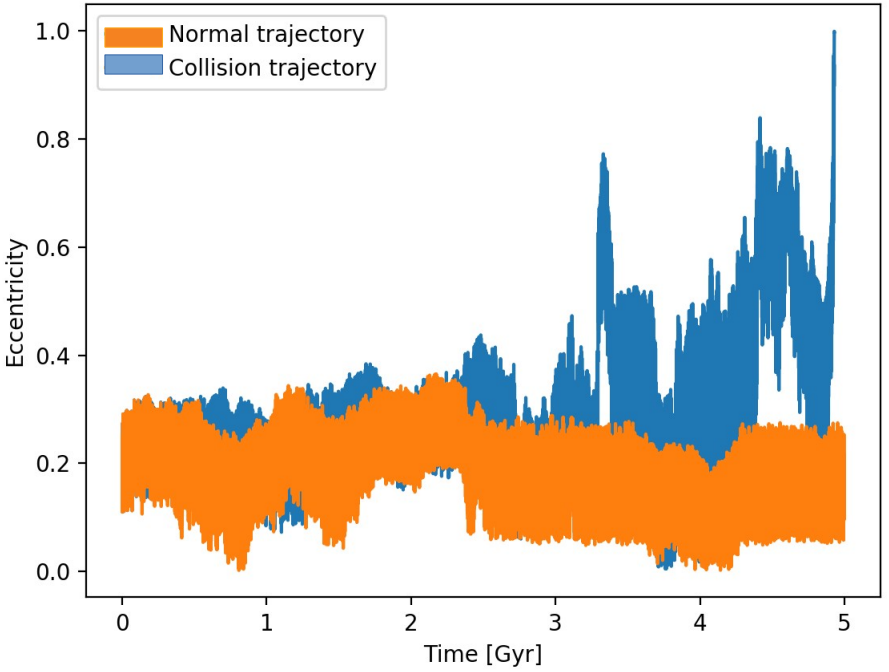
The secular model of inner Solar System

- Forced by outer planets whose trajectories are pre-determined to be quasi-periodic
- Of order 4 and order 6 in eccentricity and inclination
- Integrated from slightly-different initial conditions to produce 1000 different solutions.

Ref: Talk of F. Mogavero

Example of ejection of mercury

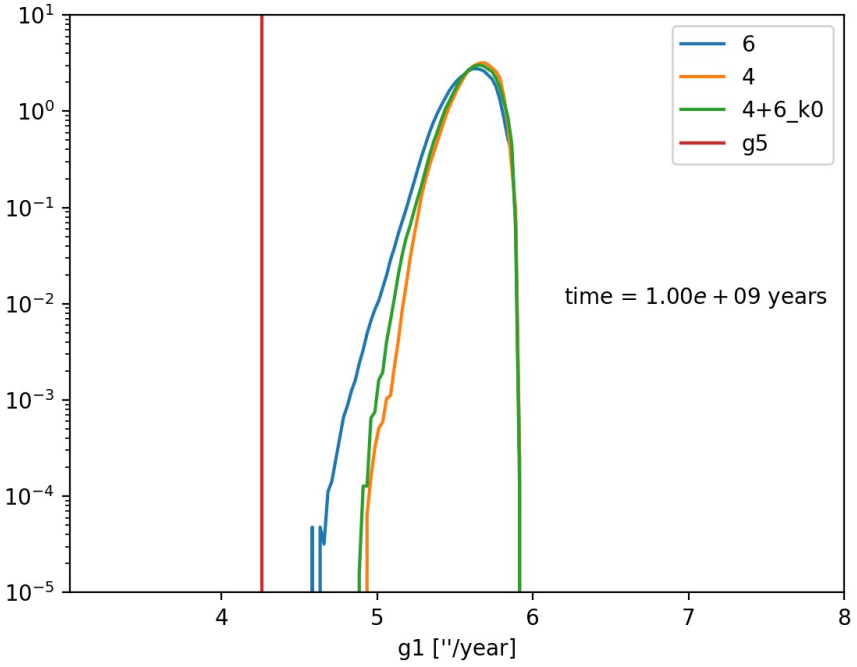


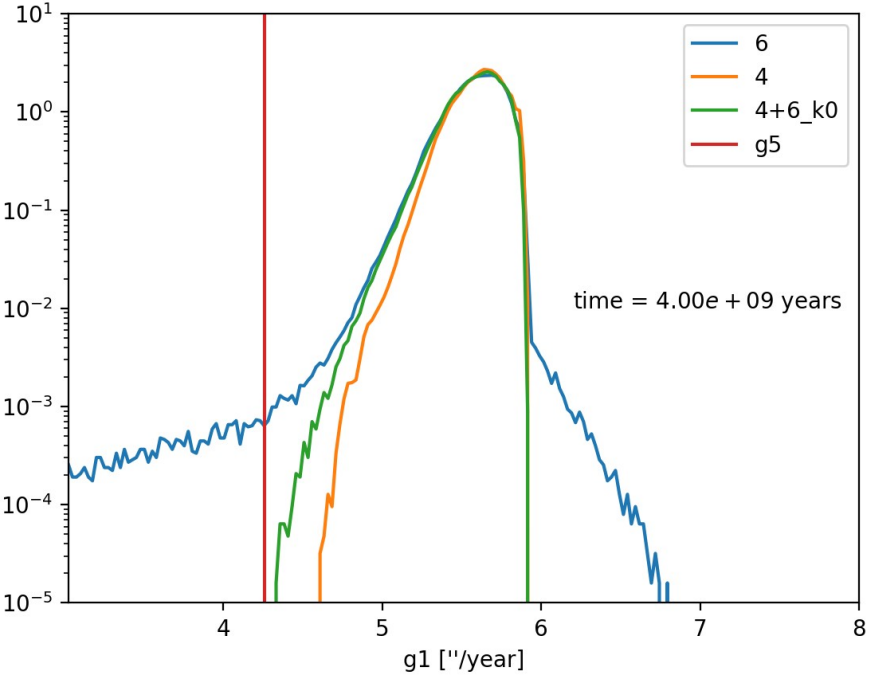


Large eccentricity of Mercury when g_1 reaches g_5

Comparison between order 4 and order 6

PDF of g_1 of complete system at order 4 and at order 6





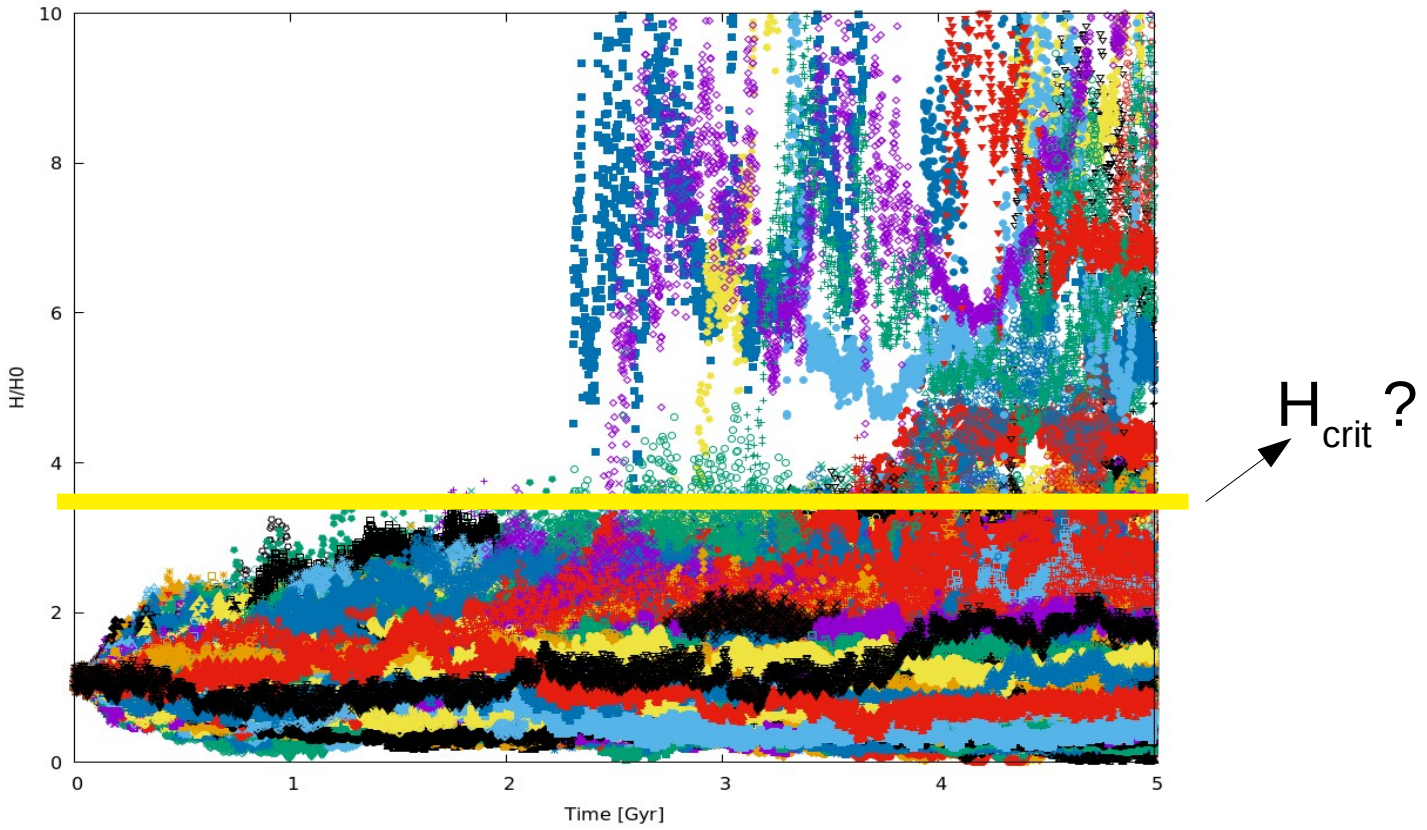
Comparison between order 4 and order 6

Number of solutions out of 1000, that have eccentricity is above a specified value

$e_{mercury}$	0.35	0.4	0.5	0.6	0.7	0.8	0.9
Order 4	331	112	4	1	0	0	0
Order 6	539	275	32	14	12	8	6
N-body integration*	492	255	40	10	9	8	8

(*from Laskar and Gastineau 2009)

Discussion

- It’s clear how Mercury is destabilised when $g_1 = g_5$
- But it’s not clear how to get there
  i.e. Simple model could explain why $T_{destabilasation} >> T_{lyapunov}$ but fails to give the correct timescale of ejection.
- Small change of Hamiltonian could result in significant difference in statistics (Statistics of Hamiltonian of order 4 and of order 6 are different)
- A semi-analytical study on the full system needs to be done.

Evolution of energy of 1000 solutions