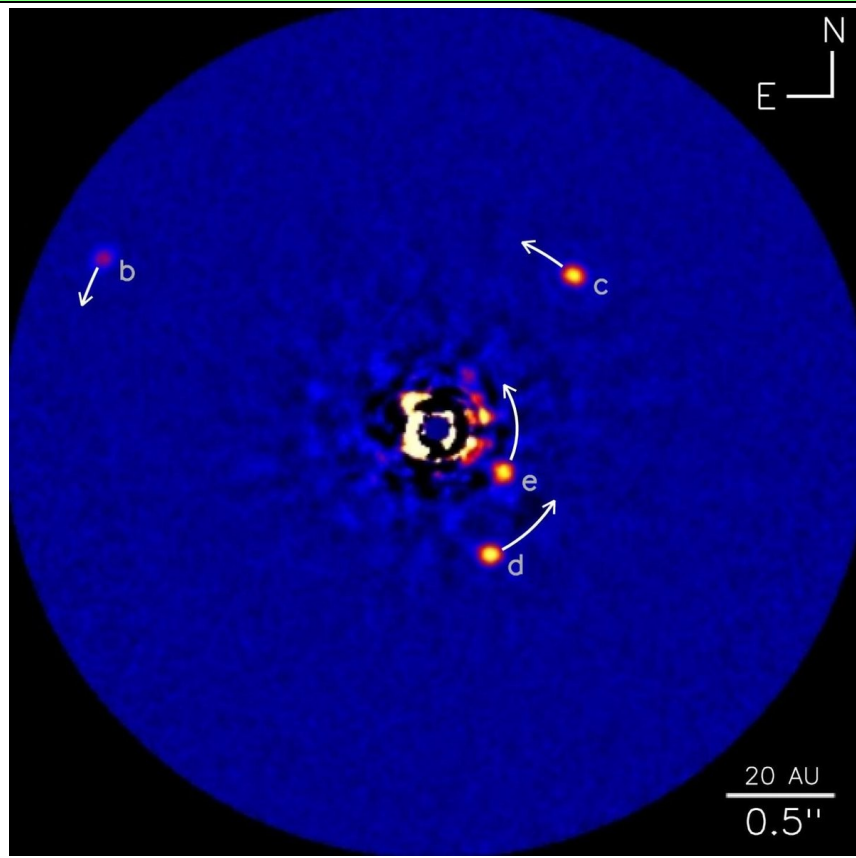


EXOPLANETS: Resonant chains & Apsidal Corotations



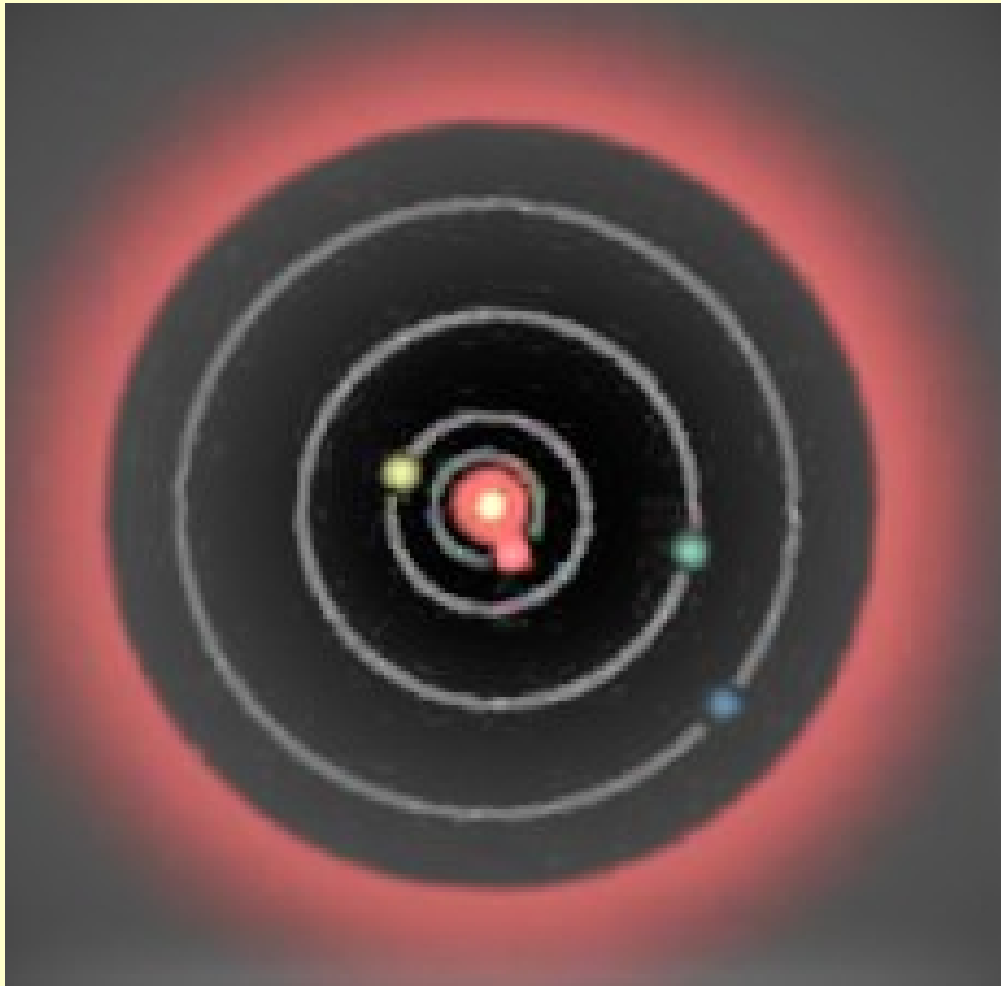
Sylvio Ferraz Mello

**Instituto de Astronomia,
Geofísica e Ciências
Atmosféricas**

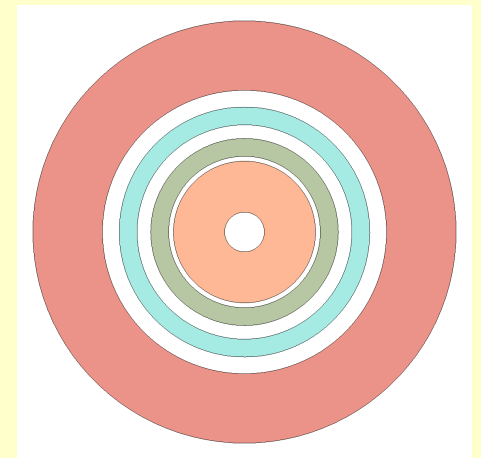
Universidade de São Paulo

June 2021

SOLAR SYSTEM ARCHITECTURE AND STABILITY



**AGE
5 BILLION
YEARS**

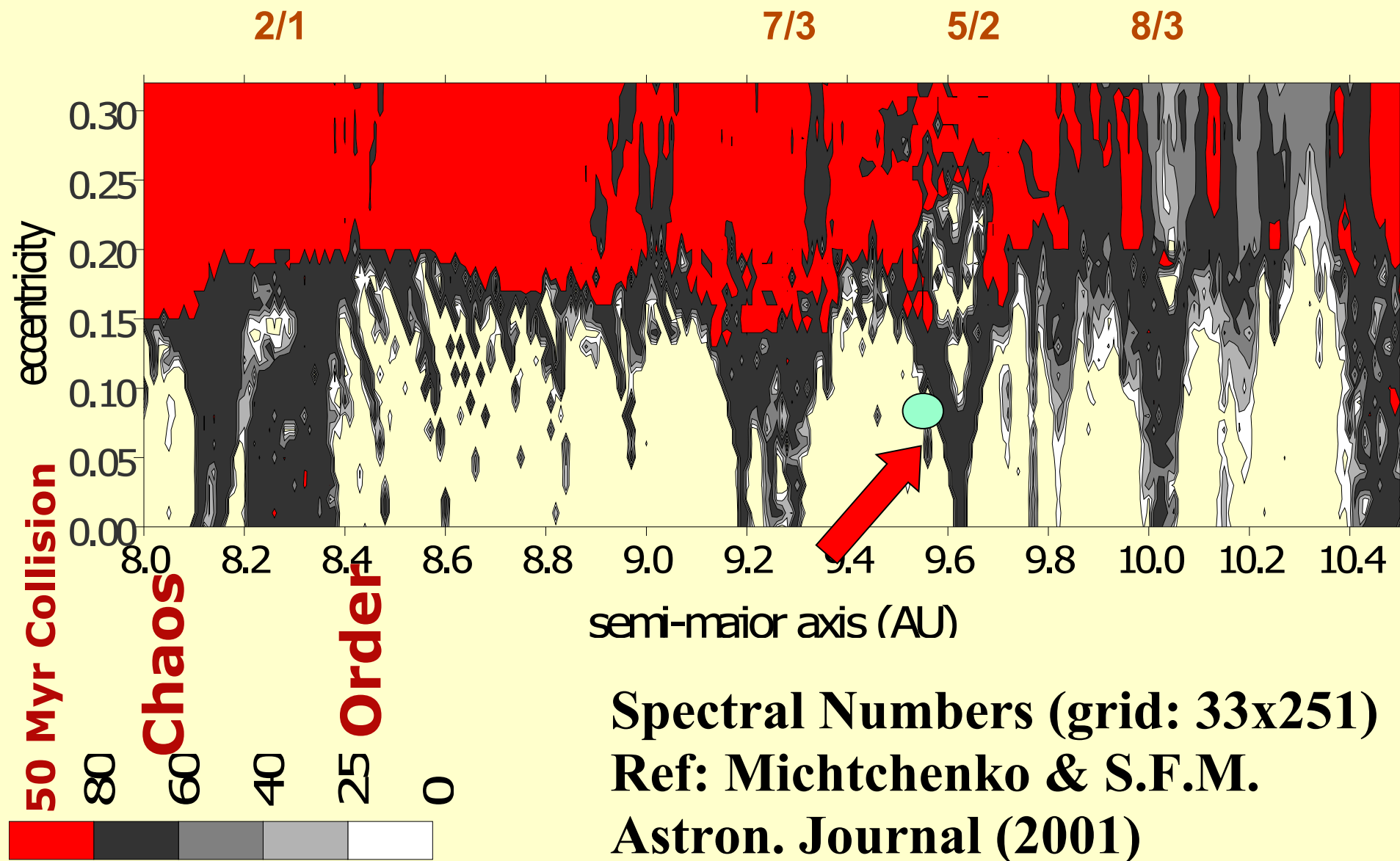


OUTER PLANETS PERIODS

		(k/k')
JUPITER	P=5.20 yr	} 2/5
SATURN	P=9.58	
URANUS	P=19.20	} 1/2
NEPTUNE	P=30.05	

$kn-k'n'$ { 0.41 deg/yr
0.63
0.08 resp.

Solar System with Saturn initialized on a grid of different initial conditions



EXOPLANETS

K2-138

			(k/k')	$2n-3n'$
I	P=2.3531 days	}	2/3	2.613 deg/d
II	P=3.5600			2.482
III	P=5.4048			2.487
IV	P=8.2616			2.496
V	P=12.7576			

This system is very close to a periodic solution

Resonant chains with 3 or more exoplanets

GJ 876 (←→)	4 (1+3)	[2:1],[2:1]	(*)
HD 158259	5	[3:2],[3:2],[3:2],[3:2]	
HR 8799 (←→)	4 (1+3)	[2:1],[2:1]	(*)
Kepler-60	3	[4:3],[5:4]	(*)
Kepler-80	5 (1+4)	[3:2],[3:2],[4:3]	(*)
Kepler-90 (duas cadeias)	8 (3+5)	[5:4],[5:3]	
		[3:2],[4:3],[3:2],[11:7]	
Kepler-223	4	[4:3],[3:2],[4:3]	
K2-138	6 (5+1)	[3:2],[3:2],[3:2],[3:2]	(*)
TOI-178 (←→)	6 (1+5)	[2:1],[3:2],[3:2],[4:3]	(*)
Trappist-1	7 (6+1)	[8:5],[5:3],[3:2],[3:2],[4:3]	(*)
V 1298 Tauri	4 (1+3)	[3:2],[2:1]	

(*) with known masses

EXOPLANETS

GJ-876

(k/k')

I P=1.94 days m=0.02 Mjupiter

II P=30.097 m=0.83

III P=61.106 m=2.66

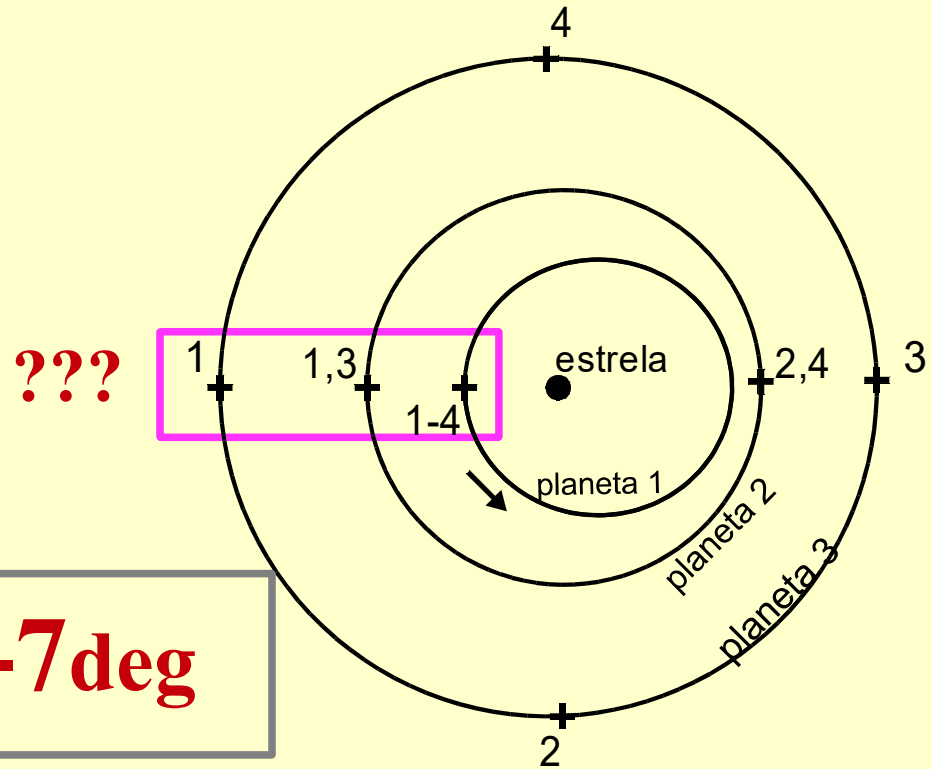
IV P=123.83 m=0.05

} 1/2

} 1/2

Laplacian
Resonance $n_1 - 3n_2 + 2n_3 = 0$

**PERIODIC
SOLUTION**
named after
DE SITTER
(Kaptein Lab
Groningen, 1907)



$$\lambda_1 - 3\lambda_2 + 2\lambda_3 = -7\text{deg}$$

different from Laplace's solution

Ref:

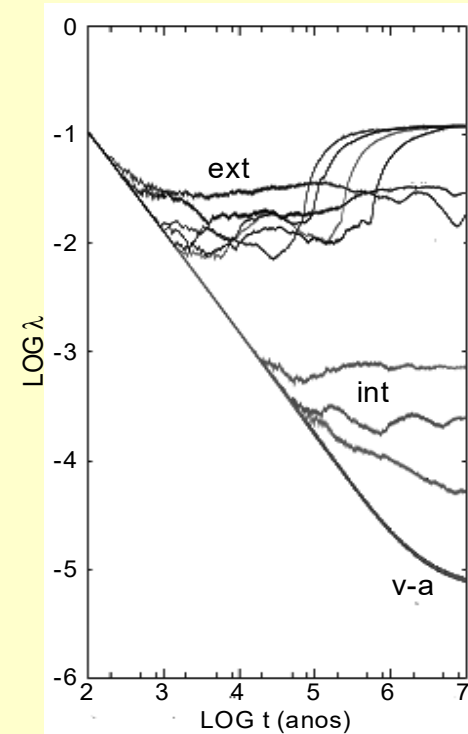
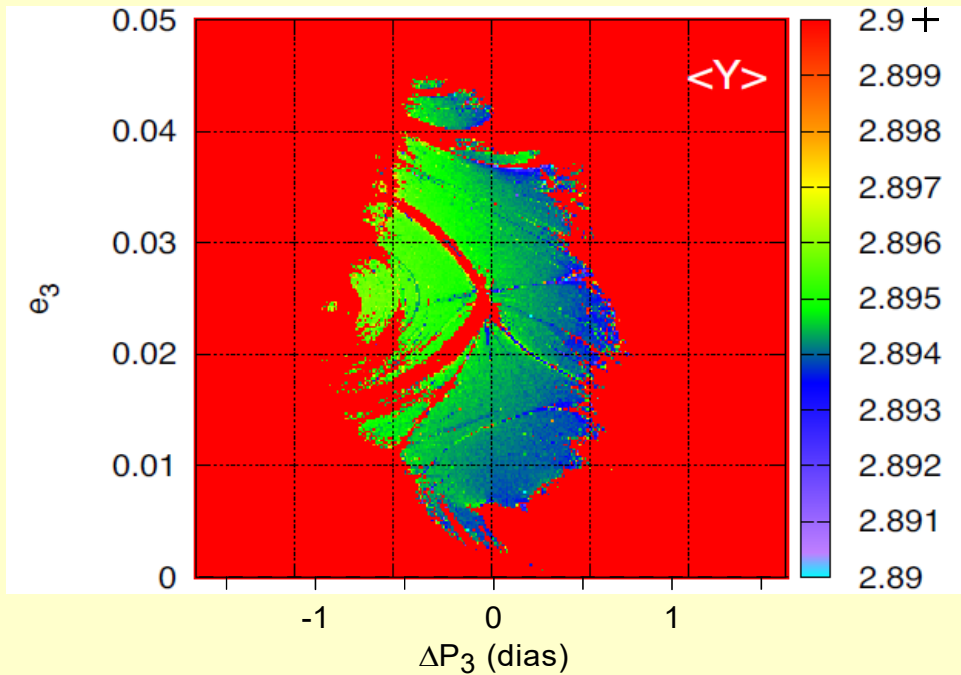
Broer, H., Hanssmann, H., Discr.Cont.Dyn.Syst. (2020)

A. Celletti et al. Celest. Mech. Dyn. Astron. (2018) 130:15

G.Pucacco, Cel. Mech, Dyn, Astron, (2021) 133:11

Lyapunov

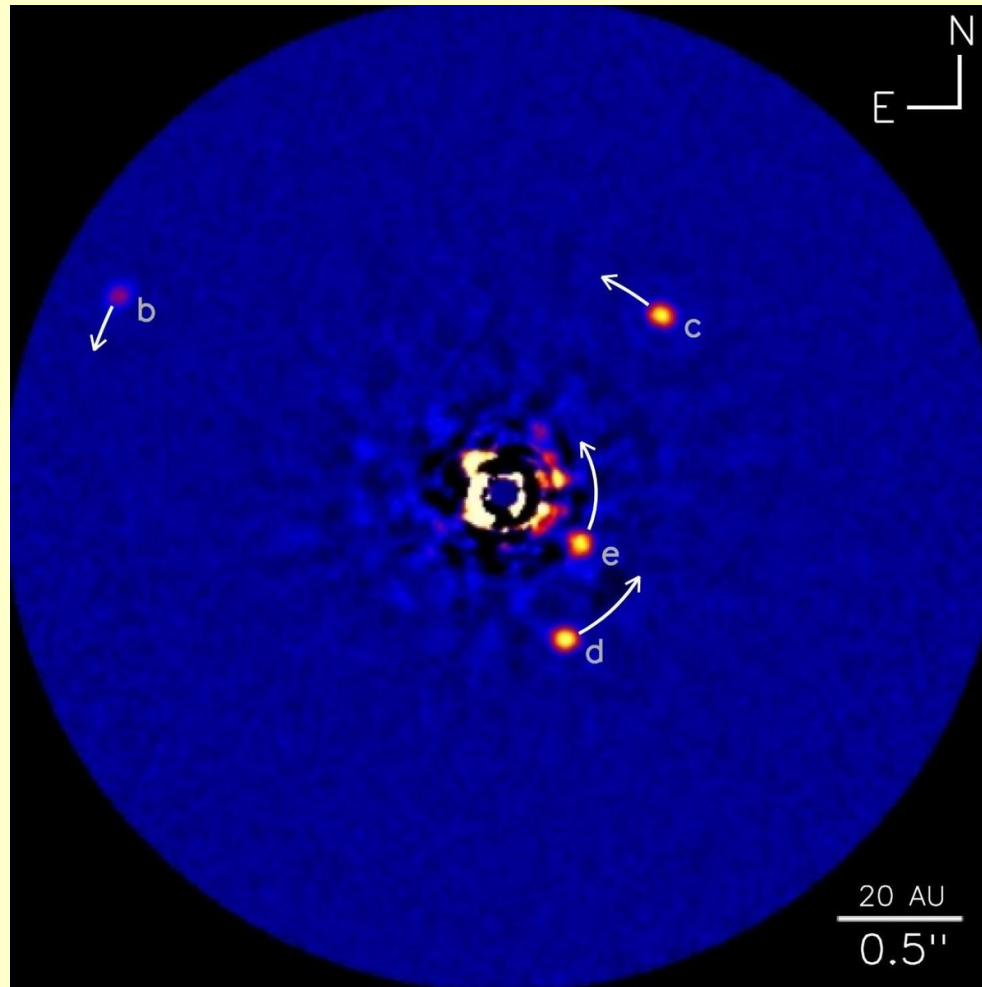
Dynamical Map with MEGNO



Ref: Martí, J.G., Cincotta, P.M., Beaugé, C.:
Chaotic diffusion in the Gliese-876 planetary system.
Mon. Not. R. Astron. Soc. 460, 1094–1105 (2016)
N.B. $\Delta P_3 \sim 2$ days

HR 8799

Age 30 Myrs



EXOPLANETS

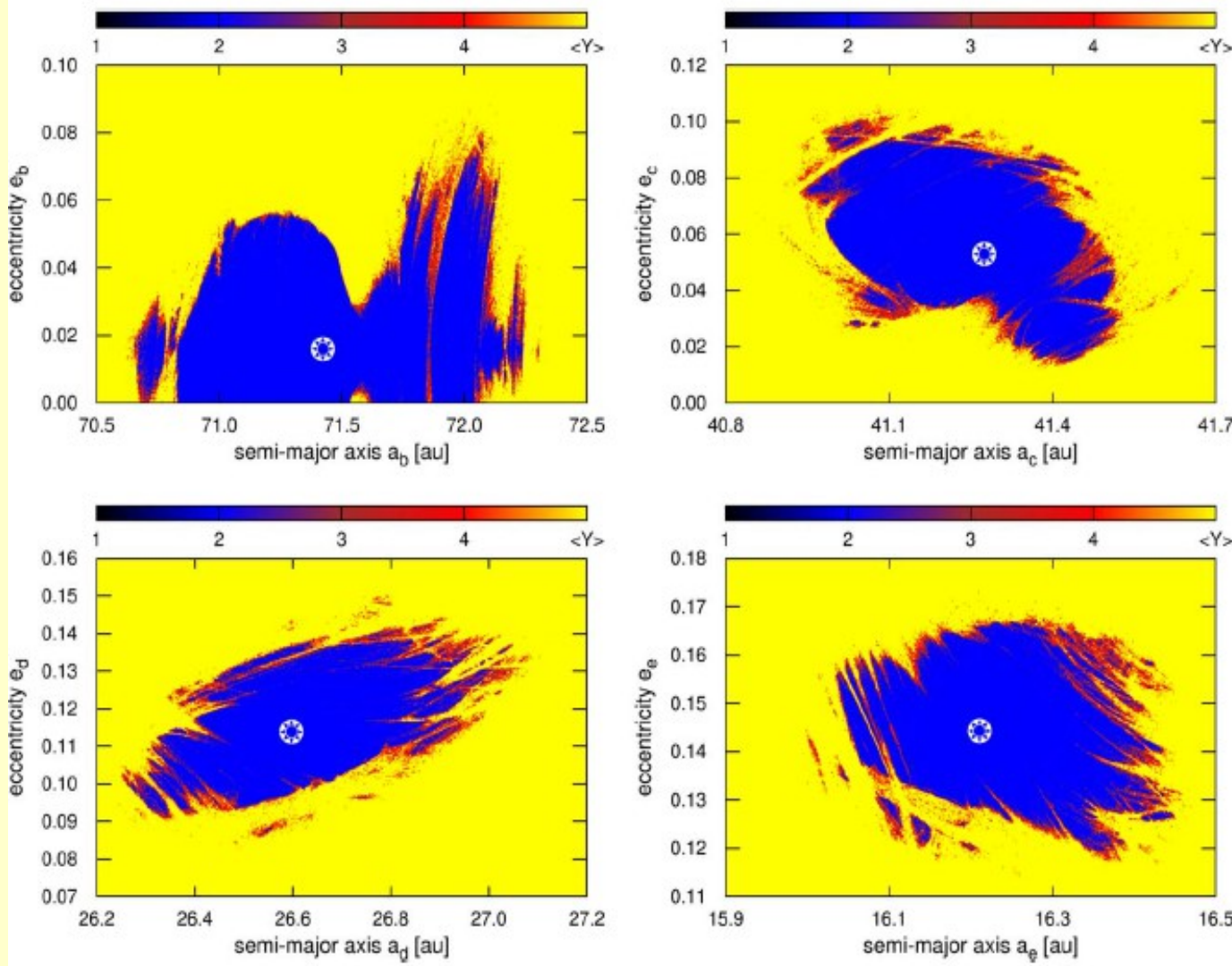
HR 8799

		(k/k')	
I	P=42 years	} $\sim 1/2$	m ~ 7 Mjup
II	P=101		} $\sim 1/2$
III	P=205	} $\sim 1/2$	
IV	P=431		m ~ 8.3

(Fabricky et al.
coplanar model)

(Goździewski &
Migaszewski)

Dynamical maps with near-periodic solution of the 1+4-body problem (Goździewski and Migaszewski *Astrophys.J.Lett.* 902:L40)



* constrained best-fit

(MEGNO)

EXOPLANETS

TOI-178 (Tess)

(A compact system
of super-Earths)

		(k/k')	kn-k'n'
I	P=1.91456 days	} 3/5	
II	P=3.23845		
III	P=6.5577	} 1/2	1.473
IV	P=9.9619	} 2/3	1.381
V	P=15.2319	} 2/3	1.372
VI	P=20.7095	} 3/4	1.370

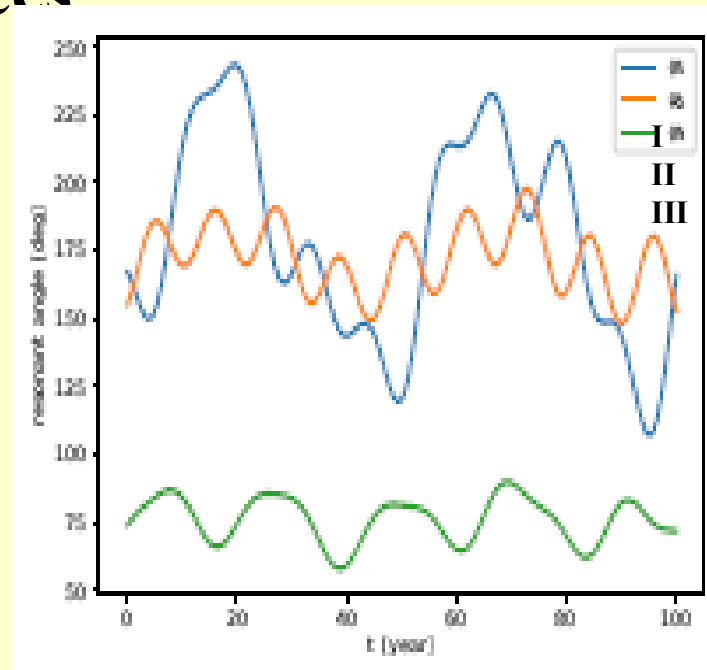
Masses: 4 to 8 earth

Planets II to VI - Chain of 3 Laplacian Resonances

$$\text{I) } n_2 - 3n_3 + 2n_4 = 0$$

$$\text{II) } 2n_3 - 5n_4 + 3n_5 = 0$$

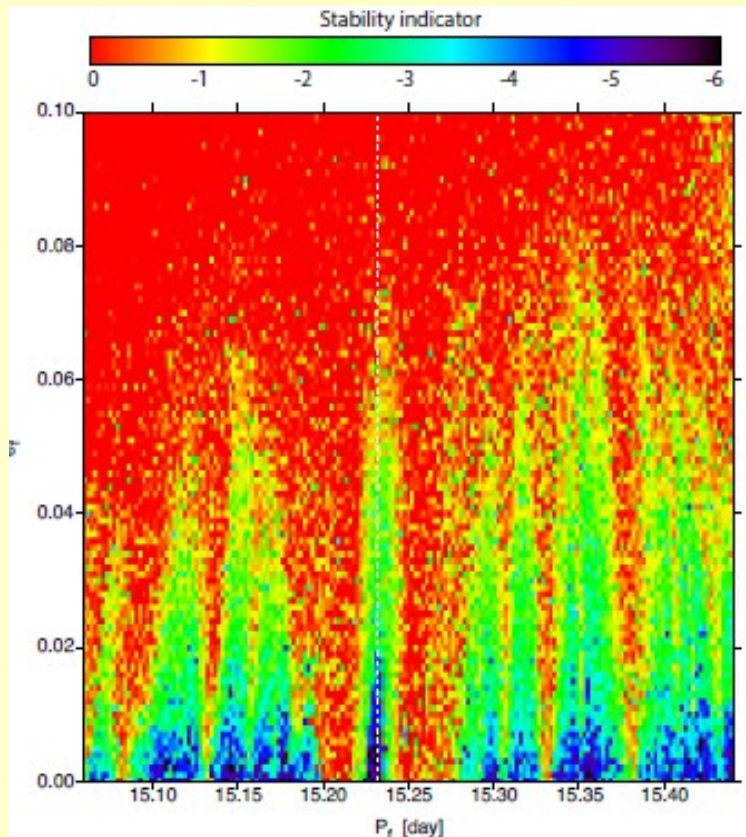
$$\text{III) (2x) } (n_1 - 3n_2 + 2n_3) = 0$$



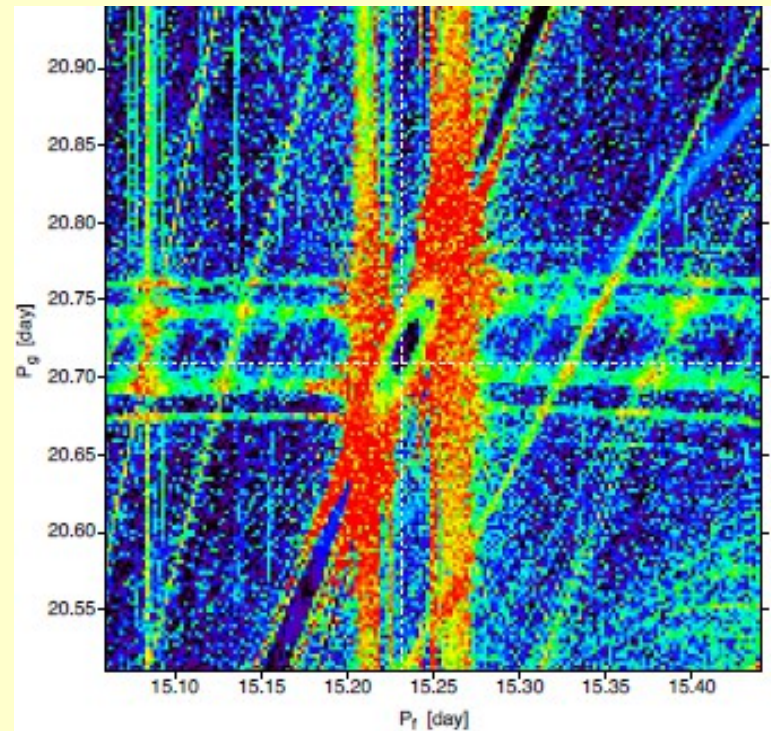
Ref: Leleu, A., et al.
Six transiting planets and a
chain of Laplace resonances in
TOI-178. *Astron. Astrophys.*,
(2021).

**Evolution of the
corresponding angles in
simulations using initial
conditions & masses as
determined from the
observations**

FREQUENCY MAP analyses of the TOI-178 resonances



P_5 vs. e_5



P_5 vs P_6

Leleu et al. 2021 (+IMCCE)

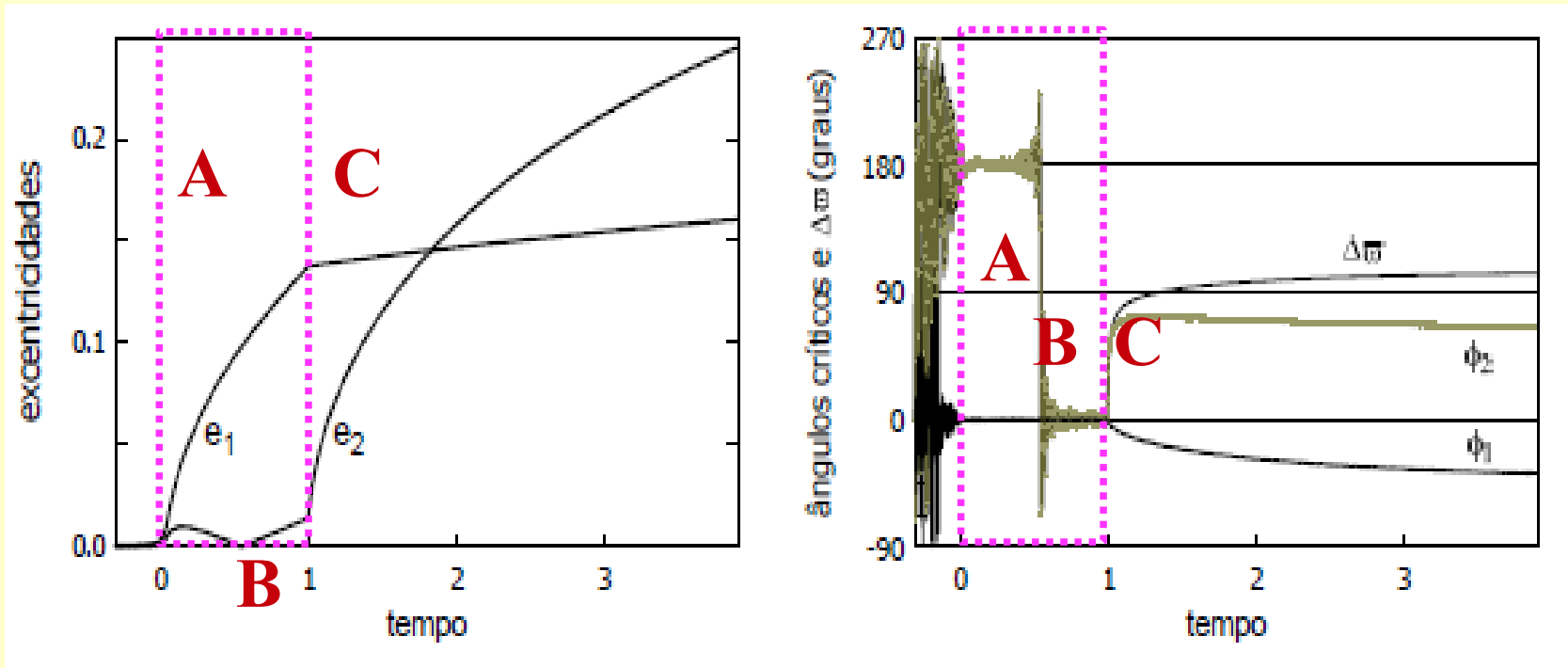
APSIDAL COROTATION RESONANCES

**Periodic solutions of the planetary 3-body problem
with $e > 0$**

APPROACHES

- 1) Periodic Orbits of the 1+2 problem –
John Hadjidemetriou and Thessaloniki school**
- 2) Numerical constructions with an ad-hoc dissipation
S. Ferraz-Mello @ São Paulo**
- 3) Equilibrium solutions of the averaged Hamiltonian
Cristian Beaugé (Cordoba, Argentina) and
Tatiana A.Michtchenko (São Paulo)**

Resonance 2:1 - Numerical constructions with an ad-hoc dissipation pushing the inner orbit outwards (convergent migration)



Events: **A**... it reaches the PO close neighborhood.

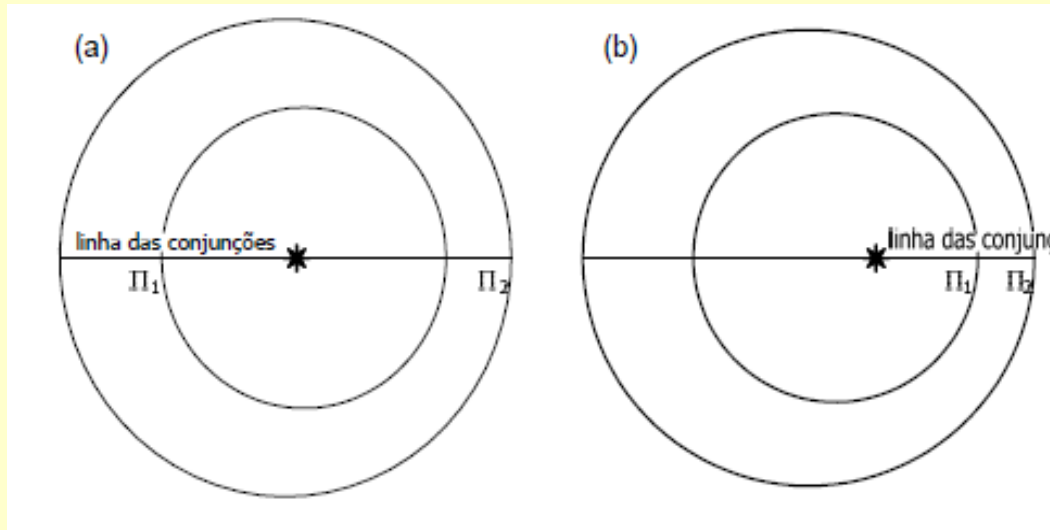
B... the pericenter of P₂ changes of side.

C... the PO becomes asymmetric

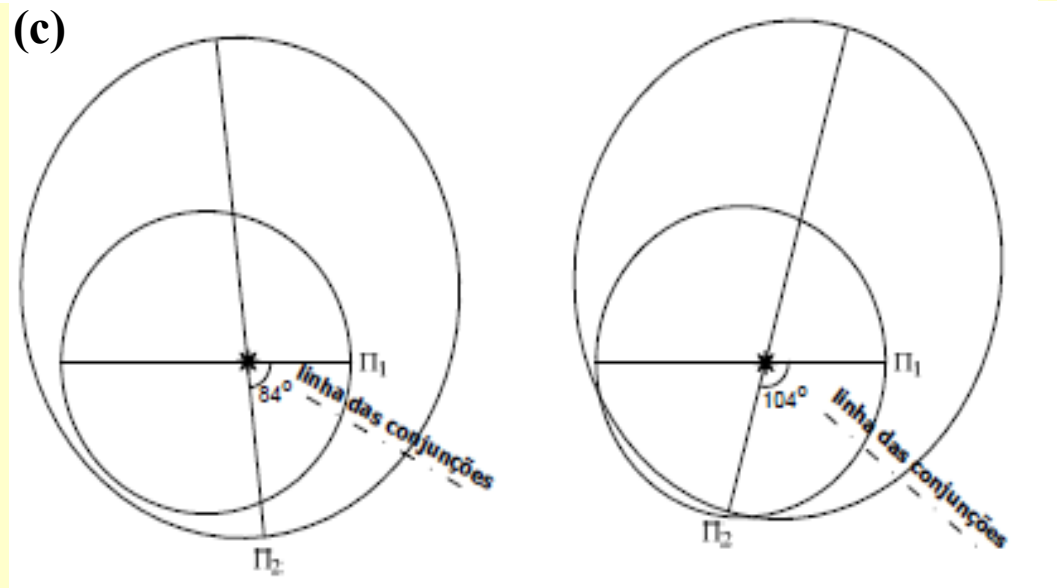
Critical angles: $\phi_j = \lambda_1 - 2\lambda_2 + \varpi_j$

Ref: **Ferraz-Mello, Beaugé and Michtchenko, CeMDA 87, 99 (2003)**

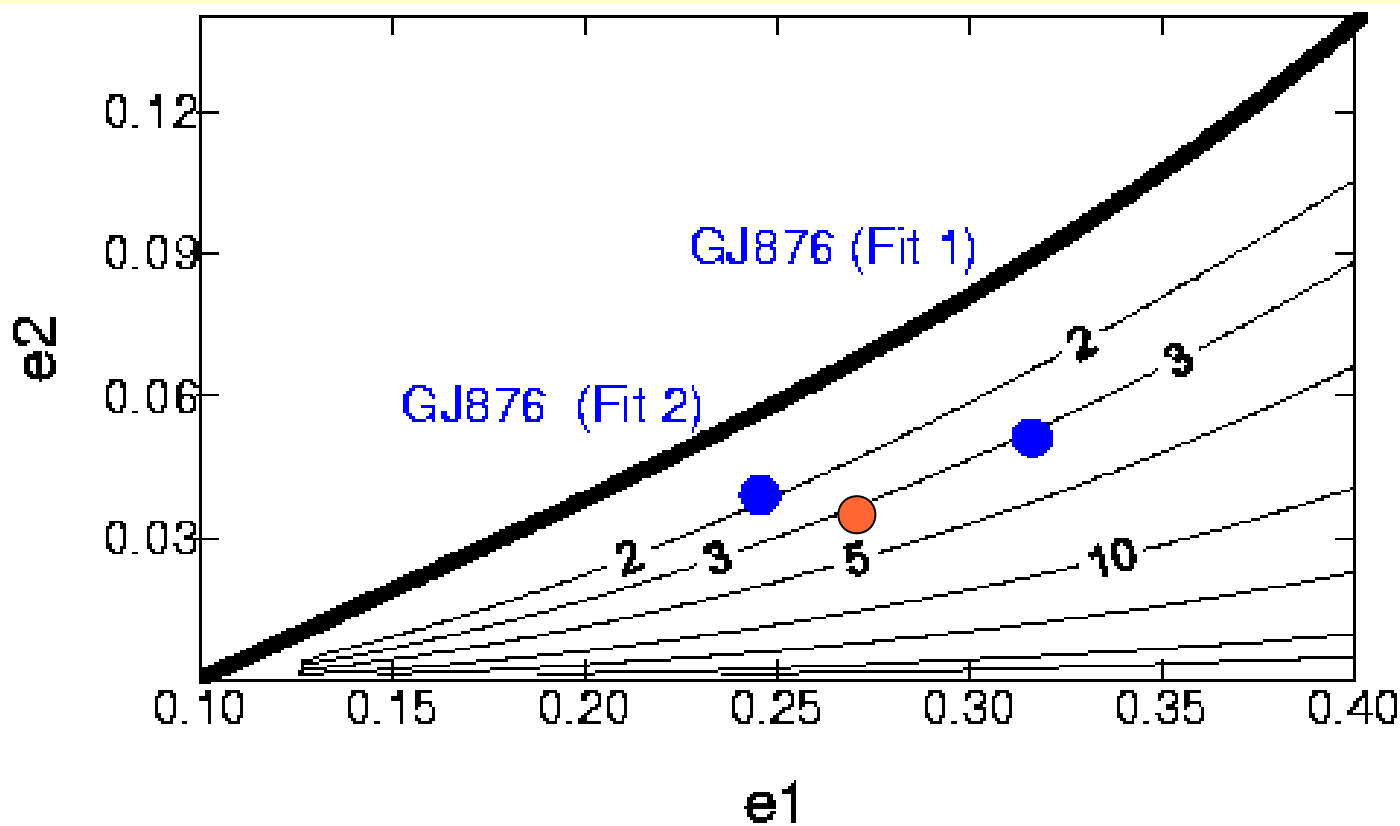
Symmetric 2/1ACR



Assymmetric 2/1 ACR



Locus of Stable stationary solutions with **aligned** periastra for different mass ratios

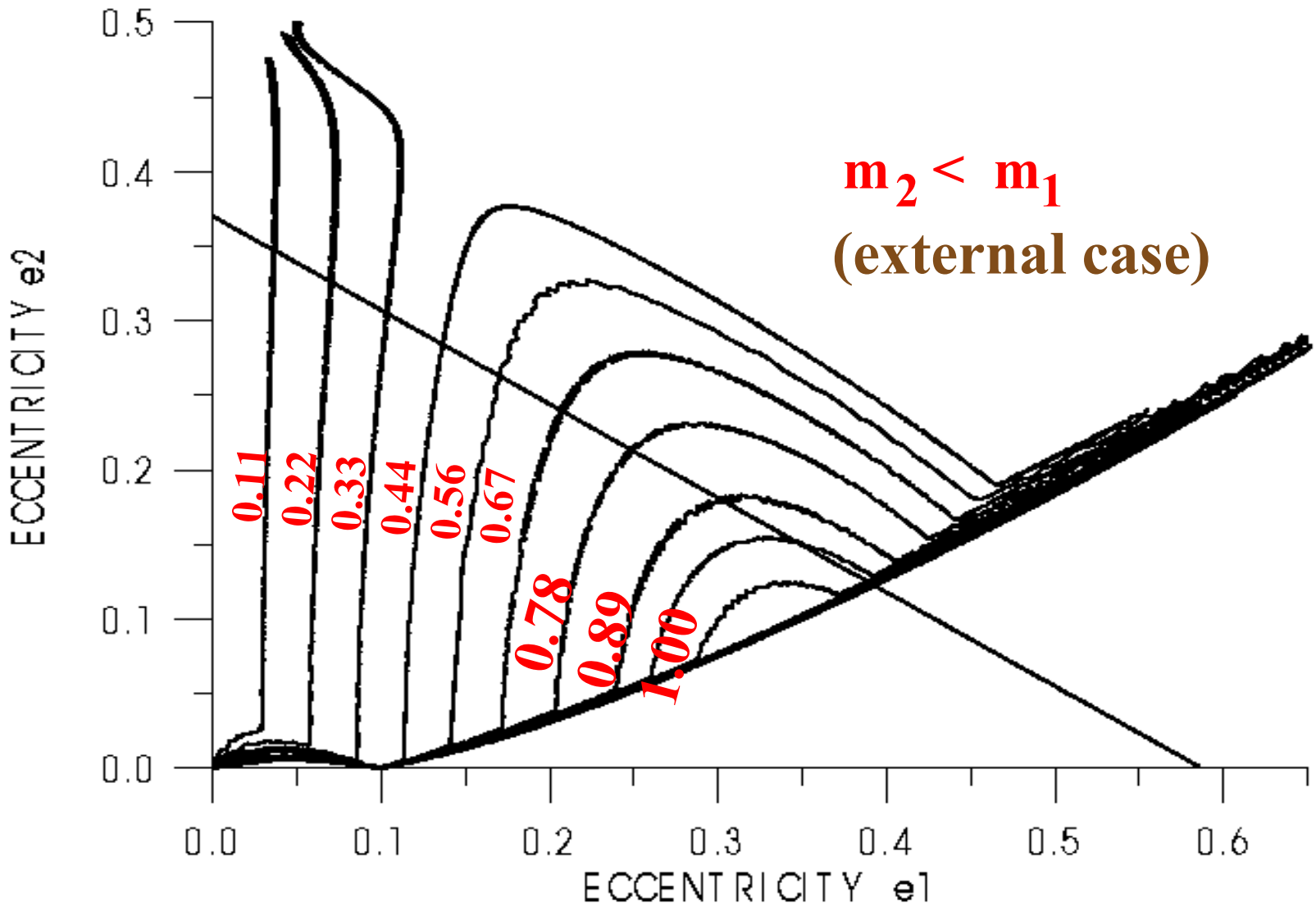


Ex: ●
GJ 876
(II,III)
 $m_2/m_1=3.02$
 $e_1=0.03$
 $e_2=0.27$
● **old**
determ.

Mass ratios $m_2/m_1 > 1$ (so-called “**internal case**”)

Ref: **Beaugé et al. (2003) Astrophys. J.**

Stable asymmetric solutions. 2/1 resonance



Ref: SFM, [Beaugé & Michtchenko, Cel. Mech. Dyn. Astron. \(2003\)](#)

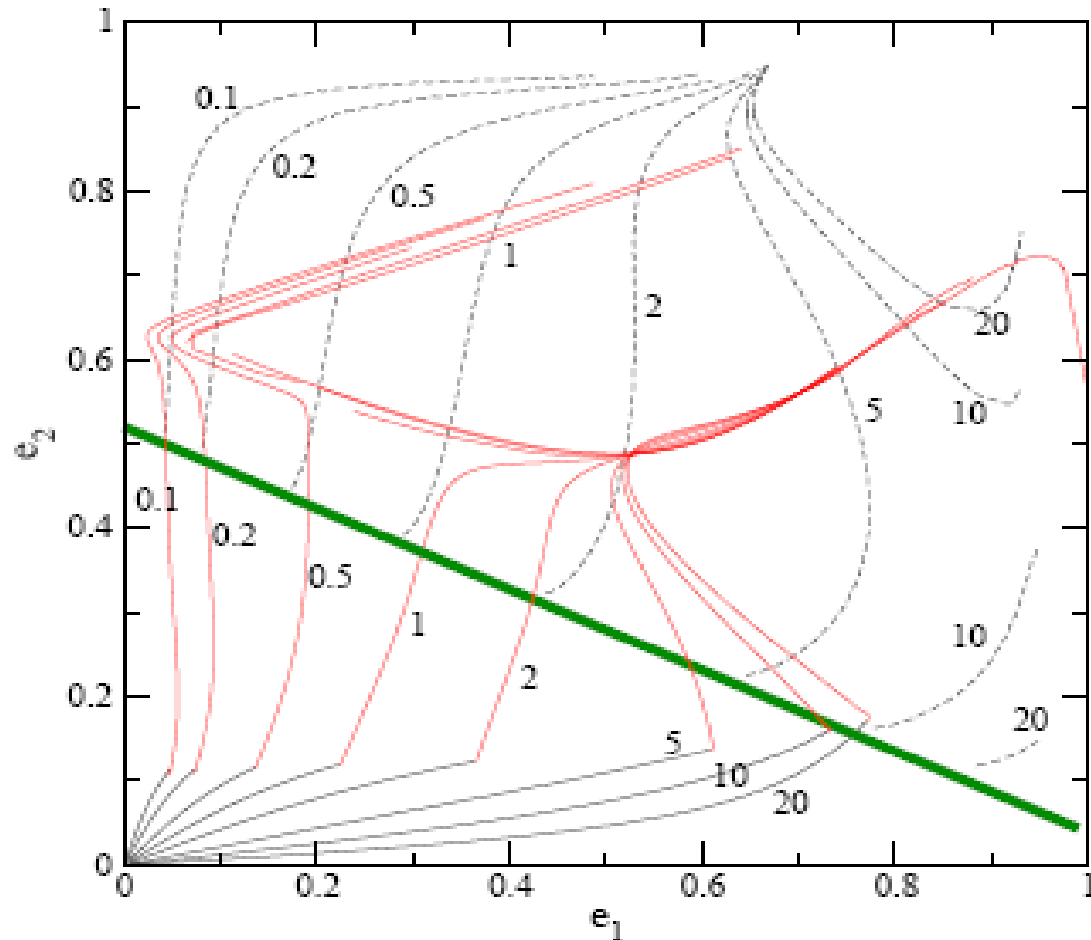


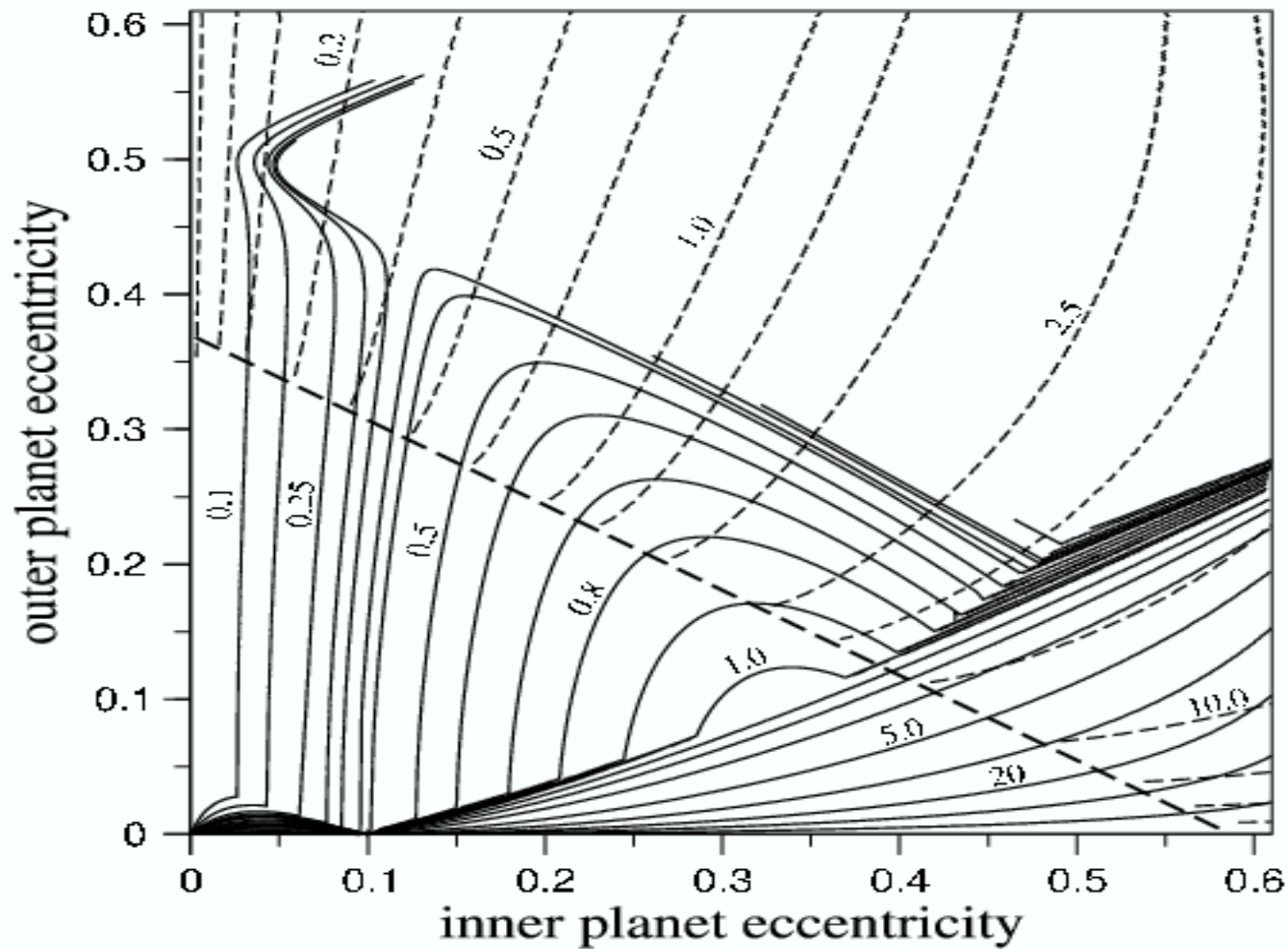
Figure 10. Families of symmetric and asymmetric apsidal corotations for the 3/1 MMR in the plane (e_1, e_2) . Black continuous lines correspond to $(0, \pi)$ solutions, red to asymmetric corotations, and broken black to (π, π) . The broad green dotted curve are points of possible collisions between the planets.

3/1 ACR

Ref. T.A.Michtchenko et al. (2006) CeMDA

end

THANKS EVERYBODY!



**Ref. T.A.Michtchenko et al. (2006) CeMDA
2/1 planetary ACR**

JUPITER'S GIANT SATELLITES (GALILEAN SATELLITES)

(k/k') n-2n'

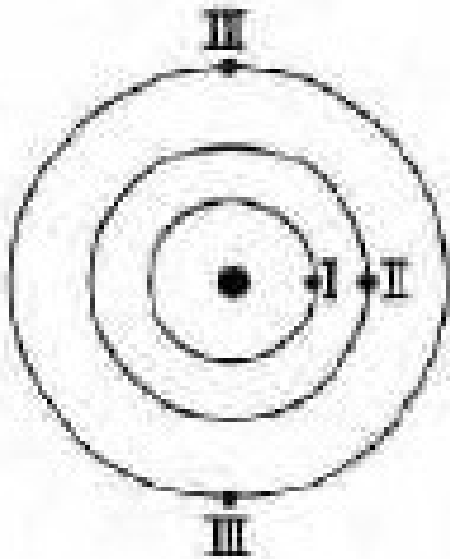
I	P= 1.77 days	}	1/2	0.740°/d
II	P= 3.55			
III	P= 7.15	}	1/2	0.739°/d
IV	P=16.7			

$$n_1 - 3n_2 + 2n_3 = 0$$

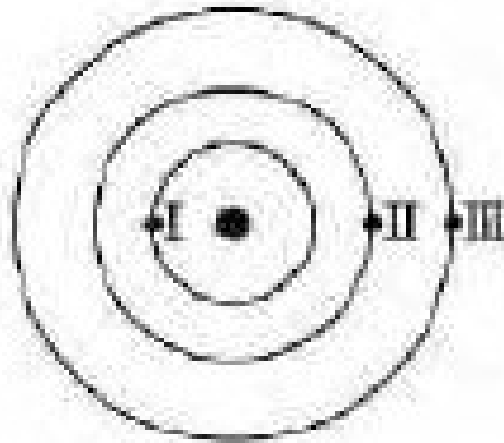
Laplacian Resonance

$$n_1 - 3n_2 + 2n_3 = 0$$

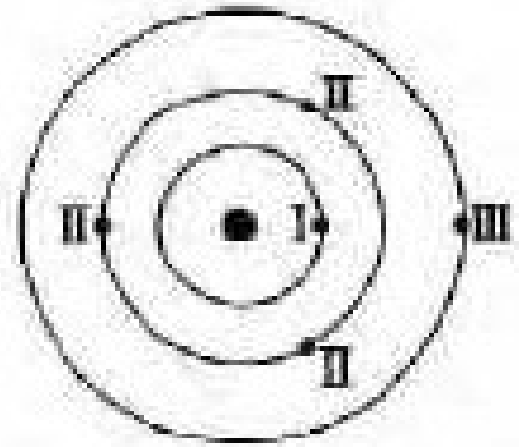
$$\lambda_1 - 3\lambda_2 + 2\lambda_3 \sim \pi$$



(a)



(b)



(c)

