Spring School

HOLOMORPHIC SYMPLECTIC MANIFOLDS AND DERIVED CATEGORIES

Palazzo Feltrinelli - Gargnano del Garda (Italy)

March 25-30, 2008

PROGRAM OF THE LECTURES

Arnaud Beauville
(Université de Nice)

Lecture 1: Manifolds with $c_1=0$

The decomposition theorem: idea of proof. Calabi-Yau manifolds; holomorphic symplectic manifolds.

Lecture 2: The classical examples

The Hilbert scheme $S^{[r]}$ of a K3 surface, the generalized Kummer variety K_r, their deformations.

Lecture 3: The period map

Definition. The standard quadratic form and its properties; the Fujiki constant.
Discussion of the surjectivity and injectivity of the period map.

Lecture 4: Birational symplectic manifolds

Mukai's symplectic flops. Huybrechts' theorem.

Lecture 5: Further developments

Cohomology: the Bogomolov-Verbitzky theorem. Restrictions on symplectic fourfolds. Lagrangian fibrations, etc.

Short list of references (more during the school):

- D. Huybrechts, *Compact hyperkähler manifolds,* in: Calabi-Yau manifolds and related geometries (Nordfjordeid, 2001), 161-225, Universitext, Springer, Berlin (2003).

- A. Beauville, *Riemannian Holonomy and Algebraic Geometry*. Enseign. Math **53** (2007), 97-126. It can be downloaded as [81] from: http://math.unice.fr/~beauvill/bibli.html

Tom Bridgeland
(University of Sheffield)

Lecture 1: Derived and triangulated categories I

Lecture 2: Derived and triangulated categories II

Lecture 3: t-structures and tilting

Lecture 4: Stability conditions

Lecture 5: Stability conditions on K3s

Daniel Huybrechts
(Universität Bonn)

Lecture 1: Generalized K3s

Lecture 2: Fourier--Mukai functors, Seidel-Thomas twists

Lecture 3: Derived Torelli

Lecture 4: Autoequivalences of K3s

Lecture 5: Some comments on mirror symmetry for K3s

Kieran O'Grady
(Università di Roma "La Sapienza")

Lecture 1: A tour through K3 surfaces

(1) Definition of a K3. Projective K3's of low degree, Kummer surfaces, K3 surfaces with du Val singularities.
(2) Cohomology of a K3, local period map and uniqueness of deformation class.
(3) Global period map for K3 surfaces. The Torelli Theorem and surjectivity of the period map. Moduli of projective K3's.

Lecture 2: First examples of higher dimensional irreducible symplectic manifolds

(1) The Beauville-Bogomolov quadratic form and Fujiki's constant of irreducible symplectic manifolds of the standard series.
(2) The Fano variety of lines on a cubic 4-fold.

Lecture 3: Moduli of sheaves on a projective Calabi-Yau surface, 1

(1) Smoothness of the moduli space. The Mukai lattice. A symplectic form on the moduli space.
(2) The case when the polarization is suitable (Yoshioka et alia). Explicit examples.

Lecture 4: Moduli of sheaves on a projective Calabi-Yau surface, 2

(1) The case when the polarization is not suitable. New irreducible symplectic varieties in dimensions 6 and 10; computation of b_2, the Beauville-Bogomolov quadratic form and Fujiki's constant.
(2) The case when the polarization is not suitable: negative results of Lehn-Sorger.

Lecture 5: Double EPW-sextics: moduli and periods

(1) Irreducible symplectic 4-folds numerically equivalent to $(K3)^{[2]}$. A plan for proving uniqueness of deformation class and a birational global Torelli Theorem.
(2) Double EPW-sextics and their periods.