

# WORKSHOP IN ALGEBRAIC GEOMETRY

Seminario di Natale 2011 - December 19-20 2011

## SCHEDULE

### December 19, 2011

11:30-12:20 Emanuele Raviolo  
14:30-15:20 Giovanni Mongardi  
15:30-16:20 Lorenzo Di Biagio  
16.40-17.30 Justin Sawon

### December 20, 2011

10:00-10:50 Riccardo Brasca  
11:00-11:50 Sönke Rollenske  
14:00-14:50 Paola Comparin  
15:00-15:50 Thomas Dedieu

The workshop will take place at the Department of Mathematics (via Cesare Saldini 50, Milano). The room is Aula Dottorato (first floor).

## TITLES AND ABSTRACTS

### ***p*-adic modular forms of non-integral weight**

*Riccardo Brasca*

We recall the basic definitions of  $p$ -adic elliptic modular forms of integral weight, as sections of line bundles. We explain how to analytically deform these line bundles in order to obtain modular forms of non-integral weight. Finally, we consider the eigencurve, that parametrizes eigenforms. These constructions can be generalized to modular forms for other PEL shimura curves.

## **Van Geemen-Sarti involutions and elliptic fibrations on K3 surfaces double cover of $\mathbb{P}^2$**

*Paola Comparin*

A van Geemen-Sarti involution on a K3 surface  $X$  is the translation by a 2-torsion section of an elliptic fibration on  $X$ . It is a symplectic involution and induces a 2-isogeny of K3 surfaces.

In a joint work with A. Garbagnati, we focus on K3 surfaces that are double covering of a blow up of  $\mathbb{P}^2$  branched along rational curves and we want to classify the van Geemen-Sarti involutions on these surfaces. In order to do so, we first classify the elliptic fibrations and then we analyse whether these fibrations admit a 2-torsion section. Moreover, we show how to obtain an explicit equation for the elliptic fibration.

## **Degeneration of K3 surfaces and applications**

*Thomas Dedieu*

I shall describe various known degenerations of K3 surfaces, and how they can be used for the study of families of curves on K3 surfaces. Among the applications, one finds existence and irreducibility results. This is based on joint work with Ciro Ciliberto.

## **Asymptotic base loci on singular varieties**

*Lorenzo Di Biagio*

The study of the asymptotic behaviour of linear series has proved to be of great importance for the development of birational algebraic geometry. In this talk we will consider two asymptotic invariants associated to linear series, namely the restricted base locus and the non-nef locus. After recalling their definitions and main properties, we will discuss their relations, proving that they coincide on normal surfaces and effective KLT pairs, thus extending a result by Ein, Lazarsfeld, Mustata, Nakamaye and Popa for smooth varieties. We will also see how, improving a theorem by Russo, these ideas can be used to characterize nef and abundant divisors on effective KLT pairs by means of asymptotic

multiplier ideals. This is a joint work with S. Cacciola (University of Roma Tre).

### **Symplectic involutions on deformations of $K3^{[2]}$**

*Giovanni Mongardi*

Let  $X$  be a Hyperkähler variety deformation equivalent to the Hilbert square of points on a K3 surface and let  $f$  be an involution preserving the symplectic form. We prove that the fixed locus of  $f$  consists of 28 isolated points and 1 K3 surface, moreover the anti-invariant lattice of the induced involution on  $H^2(X, \mathbb{Z})$  is isomorphic to  $E_8(-2)$ . Finally we prove that any couple consisting of one such variety and a symplectic involution on it can be deformed into a couple consisting of the Hilbert square of a K3 surface and the involution induced by a Nikulin involution on the K3 surface.

### **Cycles in Jacobians: infinitesimal results.**

*Emanuele Raviolo*

In the first part of the talk we will study normal functions associated to the Ceresa cycles  $W(k) - W(k)^-$  of a smooth non hyperelliptic curve of genus  $g > 3$  ( $k = 2, \dots, g - 2$ ). We will show in particular how they can be recovered from the basic normal function of  $C - C^-$  and give an infinitesimal proof that they are not algebraically trivial (Ceresa theorem). In the second part we will consider the k-theoretic counterparts of Ceresa cycles for hyperelliptic curves and show that they are undecomposable. Work in progress.

### **Lagrangian fibrations on irreducible holomorphic symplectic manifolds**

*Sönke Rollenske*

Irreducible holomorphic symplectic (IHS) manifolds form an important class of manifolds with trivial canonical bundle. One fundamental aspect of their structure theory is the question whether a given IHS manifold admits a Lagrangian fibration. I will report on a joint project

with Daniel Greb and Christian Lehn investigating the following question of Beauville: if a hyperkaehler manifold contains a complex torus  $T$  as a Lagrangian submanifold, does it admit a (meromorphic) Lagrangian fibration with fibre  $T$ ? I will describe a complete positive answer to Beauville's Question for non-algebraic IHS manifolds, and give explicit necessary and sufficient conditions for a positive solution in the general case using the deformation theory of the pair  $(X, T)$ .

## **Deformations of Fourier-Mukai transforms**

*Justin Sawon*

In this talk I will describe Toda's results on deformations of the category  $Coh(X)$  of coherent sheaves on a complex manifold  $X$ . They come from deformations of  $X$  as a complex manifold, non-commutative deformations, and gerby deformations (which can all be interpreted as deformations of  $X$  as a generalized complex manifold). Toda also described how to deform Fourier-Mukai equivalences, and I will present some examples coming from mirror SYZ fibrations.