Runge-Kutta Espliciti

Heun

0	0	0
1	1	0
	1/2	1/2

Eulero Modificato

0	0	0
1/2	1/2	0
	0	1

Ralston

0	0	0
2/3	2/3	0
	1/4	3/4

?

0	0	0	0
1/3	1/3	0	0
2/3	0	2/3	0
	1/4	0	3/4

Nyström

0	0	0	0
2/3	2/3	0	0
2/3	0	2/3	0
	2/8	3/8	3/8

Runge-Kutta 3

0	0	0	0
1/2	1/2	0	0
1	-1	2	0
	1/6	4/6	1/6

Runge-Kutta 4

0	0	0	0	0
1/2	1/2	0	0	0
1/2	0	1/2	0	0
1	0	0	1	0
	1/6	1/3	1/3	1/6

Kutta

0	0	0	0	0
1/3	1/3	0	0	0
2/3	-1/3	1	0	0
1	1	-1	1	0
	1/8	3/8	3/8	1/8

Gill **se r = sqrt(2)**

0	0	0	0	0
1/2	1/2	0	0	0
1/2	(r-1)/2	(2-r)/2	0	0
1	0	-r/2	1+r/2	0
	1/6	(2-r)/6	(2+r)/6	1/6

Merson

0	0	0	0	0	0
1/3	1/3	0	0	0	0
1/3	1/6	1/6	0	0	0
1/2	1/8	0	3/8	0	0
1	1/2	0	-3/2	2	0
	1/6	0	0	2/3	1/6

Runge-Kutta diagonalmente impliciti

Trapezi

0	0	0
1	1/2	1/2
	1/2	1/2

A-Stabile di ordine 3

$\frac{3+\sqrt{3}}{6}$	$\frac{3+\sqrt{3}}{6}$	0
$\frac{3-\sqrt{3}}{6}$	$\frac{-\sqrt{3}}{3}$	$\frac{3+\sqrt{3}}{6}$
	$\frac{1}{2}$	$\frac{1}{2}$

Butcher-Lobatto di ordine 4

0	0	0	0
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	0
1	0	1	0
	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

Hammer

0	0	0
2/3	1/3	1/3
	1/4	3/4

DIRK

(1+k)/2	(1+k)/2	0	0
0.5	-k/2	(1+k)/2	0
(1-k)/2	1+k	-1-2k	(1+k)/2
	$1/6k^2$	$1-1/(3k^2)$	$1/6k^2$

Dove k può assumere i seguenti valori 2cos(10°)/sqrt(3), -2cos(50°)/sqrt(3), -2cos(70°)/sqrt(3).

Runge-Kutta Impliciti

Gauss

s = 1 Punto Medio Implicito (si può considerare un diagonalmente implicito)

1/2	1/2
	1

s = 2

$(3-\sqrt{3})/6$	1/4	$(3-2\sqrt{3})/12$
$(3+\sqrt{3})/6$	$(3+2\sqrt{3})/12$	1/4
	1/2	1/2

s = 3

$½ - \sqrt{15}/10$	5/36	$2/9-\sqrt{15}/15$	$5/36-\sqrt{15}/30$
1/2	$5/36+\sqrt{15}/24$	2/9	$5/36-\sqrt{15}/24$
$½ +\sqrt{15}/10$	$5/36+\sqrt{15}/30$	$2/9+\sqrt{15}/15$	5/36
	5/18	4/9	5/18

Radau

s = 1 Eulero Implicito (si può considerare un diagonalmente implicito)

1	1
	1

s = 2

1/3	5/12	-1/12
1	3/4	1/4
	3/4	1/4

s = 3

$(4 - \sqrt{6})/10$	$(88-7\sqrt{6})/360$	$(296-169\sqrt{6})/1800$	$(-2+3\sqrt{6})/225$
$(4 +\sqrt{6})/10$	$(296+169\sqrt{6})/1800$	$(88+7\sqrt{6})/360$	$(-2-3\sqrt{6})/225$
1	$(16-\sqrt{6})/36$	$(16+\sqrt{6})/36$	1/9
	$(16-\sqrt{6})/36$	$(16+\sqrt{6})/36$	1/9

Runge-Kutta Immersi

Eulero Esplicito-Heun

0	0	0
1	1	0
	1/2	1/2
	1	0

Dormand-Princesche 5(4)

0							
$\frac{1}{5}$	$\frac{1}{5}$						
$\frac{3}{10}$	$\frac{3}{40}$	$\frac{9}{40}$					
$\frac{4}{5}$	$\frac{44}{45}$	$-\frac{56}{15}$	$\frac{32}{9}$				
$\frac{8}{9}$	$\frac{19372}{6561}$	$-\frac{25360}{2187}$	$\frac{64448}{6561}$	$-\frac{212}{729}$			
1	$\frac{9017}{3168}$	$-\frac{355}{33}$	$\frac{46732}{5247}$	$\frac{49}{176}$	$-\frac{5103}{18656}$		
1	$\frac{35}{384}$	0	$\frac{500}{1113}$	$\frac{125}{192}$	$-\frac{2187}{6784}$	$\frac{11}{84}$	
	$\frac{35}{384}$	0	$\frac{500}{1113}$	$\frac{125}{192}$	$-\frac{2187}{6784}$	$\frac{11}{84}$	0
	$\frac{5179}{57600}$	0	$\frac{7571}{16695}$	$\frac{393}{640}$	$-\frac{92097}{339200}$	$\frac{187}{2100}$	$\frac{1}{40}$

Fehlberg 5(4)

0						
1/4	1/4					
3/8	3/32	9/32				
12/13	1932/2197	−7200/2197	7296/2197			
1	439/216	−8	3680/513	−845/4104		
1/2	-8/27	2	−3544/2565	1859/4104	−11/40	
	16/135	0	6656/12825	28561/56430	−9/50	2/55
	25/216	0	1408/2565	2197/4104	−1/5	0

0												
$\frac{1}{18}$	$\frac{1}{18}$											
$\frac{1}{12}$	$\frac{1}{48}$	$\frac{1}{16}$										
$\frac{1}{8}$	$\frac{1}{32}$	0	$\frac{3}{32}$									
$\frac{5}{16}$	$\frac{5}{16}$	0	$\frac{-75}{64}$	$\frac{75}{64}$								
$\frac{3}{8}$	$\frac{3}{80}$	0	0	$\frac{3}{16}$	$\frac{3}{20}$							
$\frac{59}{400}$	$\frac{29443841}{614563906}$	0	0	$\frac{77736538}{692538347}$	$\frac{-28693883}{1125000000}$	$\frac{23124283}{1800000000}$						
$\frac{93}{200}$	$\frac{16016141}{946692911}$	0	0	$\frac{61564180}{158732637}$	$\frac{22789713}{633445777}$	$\frac{545815736}{2771057229}$	$\frac{-180193667}{1043307555}$					
$\frac{5490023248}{9719169821}$	$\frac{39632708}{573591083}$	0	0	$\frac{-433636366}{683701615}$	$\frac{-421739975}{2616292301}$	$\frac{100302831}{723423059}$	$\frac{790204164}{839813087}$	$\frac{800635310}{3783071287}$				
$\frac{13}{20}$	$\frac{246121993}{1340847787}$	0	0	$\frac{-37695042795}{15268766246}$	$\frac{-309121744}{1061227803}$	$\frac{-12992083}{490766935}$	$\frac{6005943493}{2108947869}$	$\frac{393006217}{1396673457}$	$\frac{123872331}{1001029789}$			
$\frac{1201146811}{1299019798}$	$\frac{-1028468189}{846180014}$	0	0	$\frac{8478235783}{508512852}$	$\frac{1311729495}{1432422823}$	$\frac{-10304129995}{1701304382}$	$\frac{-48777925059}{3047939560}$	$\frac{15336726248}{1032824649}$	$\frac{-45442868181}{3398467696}$	$\frac{3065993473}{597172653}$		
1	$\frac{185892177}{718116043}$	0	0	$\frac{-3185094517}{667107341}$	$\frac{-477755414}{1098053517}$	$\frac{-703635378}{230739211}$	$\frac{5731566787}{1027545527}$	$\frac{5232866602}{850066563}$	$\frac{-4093664535}{808688257}$	$\frac{3962137247}{1805957418}$	$\frac{65686358}{487910083}$	
1	$\frac{403863854}{491063109}$	0	0	$\frac{-5068492393}{434740067}$	$\frac{-411421997}{543043805}$	$\frac{652783627}{914296604}$	$\frac{11173962825}{925320556}$	$\frac{-13158990841}{6184727034}$	$\frac{3936647629}{1978049680}$	$\frac{-160528059}{685178525}$	$\frac{248638103}{1413531060}$	0
	$\frac{14005451}{335480064}$	0	0	0	0	$\frac{-59238493}{1068277825}$	$\frac{181606767}{758867731}$	$\frac{561292985}{797845732}$	$\frac{-1041891430}{1371343529}$	$\frac{760417239}{1151165299}$	$\frac{118820643}{751138087}$	$\frac{-528747749}{2220607170}$
	$\frac{13451932}{455176623}$	0	0	0	0	$\frac{-808719846}{976000145}$	$\frac{1757004468}{5645159321}$	$\frac{656045339}{265891186}$	$\frac{-3867574721}{1518517206}$	$\frac{465885868}{322736535}$	$\frac{53011238}{667516719}$	$\frac{2}{45}$

$\frac{1}{4}$
0