ON THE HYPERBOLIC RELAXATION OF THE ONE-DIMENSIONAL CAHN-HILLIARD EQUATION

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We study an hyperbolic relaxation of the one-dimensional Cahn-Hilliard equation, which models rapid spinodal decompositions in a binary alloy. More precisely, for any fixed \( \varepsilon \in (0, 1] \), we deal with the following problem for the relative temperature \( u = u(x, t) \).

\[
\begin{align*}
\varepsilon u_{tt} + u_t - ( - u_{xx} + \phi(u) )_{xx} &= 0, & \text{in} & \ (0, \ell) \times (0, \infty), \\
u(0, t) = u(\ell, t) = u_{xx}(0, t) = u_{xx}(\ell, t) &= 0, & t & \geq 0, \\
u(x, 0) = u_0(x), & \quad x & \in (0, \ell), \\
\varepsilon u_t(x, 0) = \varepsilon u_1(x), & \quad x & \in (0, \ell),
\end{align*}
\]

where \( \phi \) is a smooth function with cubic growth. Our aim is to prove that, as \( \varepsilon \to 0 \), the asymptotic behavior of these problems converges to the longterm dynamics of the Cahn-Hilliard equation. Thus, first of all, we show that, properly choosing the phase-space, each problem generates a (dissipative) dynamical system, which admits a global attractor. Moreover, the resulting family is upper-semicontinuous at \( \varepsilon = 0 \). By the more recent concept of exponential attractor, we can improve the continuity result. Namely, taking advantage of a recent abstract theorem, we construct a robust family of exponential attractors, whose common basins of attraction are the whole phase-space. These results are contained in a joint paper with M. Grasselli, A. Miranville and V. Pata.

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