A REACTION-DIFFUSION EQUATION
WITH MEMORY

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Let $\Omega \subset \mathbb{R}^N$, $N = 1, 2, 3$, be a bounded domain. For $u = u(x, t) : \Omega \times \mathbb{R} \to \mathbb{R}$, we consider

$$u_t - \int_0^\infty k(s) \Delta u(t - s) ds + \phi(u) = f, \quad t > 0,$$

where $k : [0, \infty) \to [0, \infty)$ is a suitable memory kernel, the function $\phi$ is a smooth non-linearity with (at most) cubic growth, and $f$ is a given external force. Equation (1) for $N = 1$, endowed with homogeneous Dirichlet boundary condition, has been introduced in [5] to model the behavior of certain viscoelastic fluids (see also [2] for the case $N = 3$). There, $u$ represents the velocity of the fluid and $\phi(\xi) = \xi^3 - R\xi$, $R$ being the Rayleigh number. Moreover, this equation has been proposed in [3] as a model of further different phenomena. The well-posedness of the Cauchy-Dirichlet problem associated with (1) in a history space formulation has been recently analyzed in [1]. In the same paper, the authors have replaced $k$ by the time-rescaled kernel

$$k_\varepsilon(s) = \frac{1}{\varepsilon} k\left(\frac{s}{\varepsilon}\right), \quad \varepsilon \in (0, 1],$$

and they have analyzed the behavior of the solution as $\varepsilon$ goes to 0, showing that it suitably converges to the unique solution to the standard reaction-diffusion equation (note that $k_\varepsilon \to \delta$ in the sense of distributions, provided that the integral of $k$ over $(0, \infty)$ is equal to 1). Here I want to illustrate some recent preliminary results regarding the longterm behavior of solutions (cf. [4]). More precisely, I will show first that the same initial and boundary value problem considered in [1] generates a dissipative dynamical system on an appropriate phase-space which accounts for the past history of $u$. Then, I will present the main result, namely, the existence of a global attractor in the one-dimensional case for $\varepsilon$ sufficiently small. Finally, I will mention some related open questions.

References