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Inverse and Control Problems for PDE's (ICOP)

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ABSTRACTS BOOK

Analytic hypo-ellipticity in the presence of lower order terms

Paolo Albano

University of Bologna, Italy

It is well known that the hypoellipticity of a partial differential operator heavily depends on the lower order terms, both in the C^∞ and in the analytic category. We consider a second order operator with analytic coefficients whose principal symbol vanishes exactly of order two on a symplectic real analytic manifold. We assume that the first (non degenerate) eigenvalue vanishes on a symplectic submanifold of the characteristic manifold. In the C^∞ framework this situation would mean a loss of 3/2 derivatives. We prove that this operator is analytic hypo-elliptic. The main tool is the FBI transform. A case in which the C^∞ hypo-ellipticity fails is also discussed.

Boundary control problems for hyperbolic equations

Fabio Ancona

University of Bologna, Italy

We will discuss some results concerning the boundary controllability and stabilizzability of first-order hyperbolic equations, where the boundary data are regarded as input-controls. It will be first considered the controllability problem for a scalar hyperbolic conservation laws, and next we will address the problem of asymptotic stabilizzability to constant state for system of conservation laws. As a first step in this direction we will discuss the problem of global exact controllability for first order linear hyperbolic systems with constant coefficients. In particular, we shall consider the case where only a partial control of the boundary values is available.

On the determination of thin elastic inclusions from boundary measurements

Elena Beretta

University of Roma La Sapienza, Italy

In this talk we want to present some recent results concerning the identification of thin elastic inclusions in an elastic body from traction-displacement relations measured on the boundary of the region Ω occupied by the body. We accomplish this by deriving a rigorous asymptotic formula of the displacement field u_ϵ with respect to the thickness parameter of the inclusion ϵ as $\epsilon \to 0$.

Recent advances in the analysis of linear and nonlinear coupled PDE systems

FRANCESCA BUCCI

University of Firenze, Italy

In this lecture we wish to present some recent advances in two distinct yet related areas of the research in Partial Differential Equations (PDEs).
A first set of results will concern the regularity analysis of specific coupled PDE systems such as they arise in thermoelasticity or in fluid-structure interactions (*structural acoustic models*). The main motivation for establishing suitable sharp interior/boundary regularity estimates of the corresponding solutions comes from the study of the associated optimal quadratic control problems, where the issue of regularity of the (controlled and uncontrolled) dynamics plays a key role.
As time permits, we will next discuss the long-time behaviour of solutions to a composite nonlinear PDE model: more precisely, the issues of existence of global attractors as well as their topological/structural properties. The results achieved benefit from methods and tools pertaining to control theory, which only very recently gave rise to significant progress in the asymptotic analysis of infinite-dimensional dynamical systems, and of hyperbolic (hyperbolic-like) dynamics with nonlinear dissipation in particular.

Some global in time results for parabolic integro–differential inverse problems

FABRIZIO COLOMBO

Polytechnic of Milano, Italy

Joint work with D. Guidetti

The main new results of the talk are related to the following abstract version of a parabolic integro-differential inverse problem:

Definition 0.1. Determine $\tau \in (0, T]$ and

$$u \in W^{2,p}(0,\tau;X) \cap W^{1,p}(0,\tau;D(A)), \qquad h \in L^p(0,\tau), \quad t \in (0,\tau) \tag{0.1}$$

satisfying the system

$$\begin{cases} u'(t) = Au(t) + h * Bu(t) + f(u(t)) + G(t), & t \in (0,\tau) \\ u(0) = u_0, & \\ \Phi(u) = g(t) & t \in (0,\tau). \end{cases} \tag{0.2}$$

We solve the inverse problem under the following conditions on the data. Let $p \in (1, +\infty)$ and let X, Y, $D(A)$ be Banach spaces such that:

(H1) $D(A) \hookrightarrow Y \hookrightarrow X$, $D(A)$ is dense in X.

(H2) A is the infinitesimal generator of an analytic semigroup in X.

(H3) $B \in \mathcal{L}(D(A), X)$.

(H4) $u_0 \in D(A)$.

(H5) $\Phi \in X'$.

(H6) $f \in C^1(Y, X)$ and $f' : Y \to \mathcal{L}(Y, X)$ is Lipschitz continuous in bounded subsets of Y.

(H7) $G \in W^{1,p}(0, T; X)$.

(H8) $v_0 := Au_0 + f(u_0) + G(0) \in (X, D(A))_{1-1/p,p}$.

(H9) $\Phi(Bu_0) \neq 0$.

(H10) $g \in W^{2,p}(0, T)$ with $\Phi(u_0) = g(0)$ and $\Phi(v_0) = g'(0)$.

(H11) $f' : Y \to \mathcal{L}(Y, X)$ is bounded, with f' Frechèt derivative of f.

The main results are the following:

Theorem 0.1. (Local in time existence). Let the assumptions $(H1)$–$(H10)$ hold. Then there exists $\tau \in (0, T]$, depending on the data, such that the inverse problem in Definition 0.1 has a solution $(u, h) \in [W^{2,p}(0, \tau; X) \cap W^{1,p}(0, \tau; D(A))] \times L^p(0, \tau)$.

Theorem 0.2. (Global in time uniqueness). Let the assumptions $(H1)$–$(H10)$ hold. Then, if $\tau \in (0, T]$, and the inverse problem in Definition 0.1 has two solutions $(u_j, h_j) \in [W^{2,p}(0, \tau; X) \cap W^{1,p}(0, \tau; D(A))] \times L^p(0, \tau)$ $(j \in \{1, 2\})$, then $u_1 = u_2$ and $h_1 = h_2$.

Theorem 0.3. (Global in time existence and uniqueness). Let the assumptions $(H1)$–$(H11)$ hold. Let $T > 0$. Then the inverse problem in Definition 0.1 has a unique solution $(u, h) \in [W^{2,p}(0, T; X) \cap W^{1,p}(0, T; D(A))] \times L^p(0, T)$.

Identifications problems for parabolic equations with delay

GABRIELLA DI BLASIO

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Let A be the infinitesimal generator of an analytic semigroup in a Banach space E. We consider the problem of recovering the unknown right-hand side $f : [0, T] \to \mathbb{R}$ in the following delay functional differential initial value problem in E:

$$\begin{cases} u'(t) = Au(t) + aAu(t-r) + \int_{-r}^{0} b(s)Au(t+s)\,ds + f(t)z, & \text{for a.e. } t \in (0, T), \\ u(s) = \varphi_1(s), \ \text{for a.e. } s \in (-r, 0); \ u(0) = \varphi_0, \end{cases}$$

where $a \in \mathbb{R}$, $b : (-r, 0) \to \mathbb{R}$, $z \in E$, $\varphi_1 : (-r, 0) \to E$ and $\varphi_0 \in E$ are given.

To recover the scalar-valued function f we prescribe the additional information

$$\Phi[u(t)] = g(t), \qquad t \in [0, T],$$

where $g : [0, T] \to \mathbb{R}$ and Φ are, respectively, a given function and a linear function defined on $D(\Phi) \subseteq E$. We do not require that $D(\Phi) = E$; as an example this will allow, in the case where A arises from a second order elliptic operator on Ω, to consider also conditions involving the derivatives of u.

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An identification problem for nonhomogeneous thermal bodies made of material with memory

Alberto Favaron

University of Milano, Italy

We deal with an identification problem arising when the classical theory of heat conduction is modified in order to describe the thermal behaviour of viscoelastic materials such as polymers, polymer solutions and suspensions. With respect to metals, the mechanical properties of viscoelastic materials can be affected by the previous history, that is, by the method of fabrication, post-treatment, and the age of the finished article. This is why these materials are said to possess *memory*. Such a memory evinces in the constitutive relationship between the stress and the strain tensor and leads to mathematical models of viscoelastic phenomena which take the form of partial differential Volterra equations.

Indeed, if $\Omega \subset \mathbf{R}^3$ is a nonhomogeneous thermal body made of material with memory, then the variation of the temperature u with respect to time satisfies the following parabolic integro-differential equation, where the symbol "$*$" stands for the convolution operator $(v * w)(t) = \int_0^t v(t-s)w(s)\,\mathrm{d}s$:

$$D_t u(t,x) = \mathcal{A}u(t,x) + \operatorname{div}\Big[\Big(k(\cdot,\rho(x)) * \widetilde{b}(x)\nabla u(\cdot,x)\Big)(t)\Big] + f(t,x)\,, \quad t > 0,\ x \in \Omega. \quad (0.3)$$

Here $\mathcal{A}$ is a second-order linear differential operator, k represents the *memory kernel* keeping the history record of the material, $\rho : \Omega \to \mathbf{R}$ is an assigned function, f represents the system of external heat sources, $\widetilde{b}(x)$ denotes a 3×3 matrix $(\widetilde{b}_{i,j}(x))_{i,j=1}^3$ and, finally, $\operatorname{div} = \sum_{i=1}^3 D_{x_i}$ and $\nabla = (D_{x_1}, D_{x_2}, D_{x_3})$.

Here we show that, under suitable assumptions and two pieces of additional information, the problem of recovering the memory kernel k in equations of the type of (0.3) related to a bounded domain $\Omega \subset \mathbf{R}^3$ can be uniquely solved locally in time.

Carleman estimates for degenerate parabolic operators and null controllability

Genni Fragnelli

University of Siena, Italy

In this talk an estimate of Carleman type for the one dimensional heat equation

$$u_t - (a(x)u_x)_x + c(t,x)u = h(t,x), \quad (t,x) \in (0,T) \times (0,1),$$

is proved. Here $a(\cdot)$ is degenerate at 0. Such an estimate is derived for a special pseudo-convex weight function related to the degeneracy rate of $a(\cdot)$. Then, the null controllability on $[0,1]$ of the semilinear degenerate parabolic equation

$$u_t - (a(x)u_x)_x + f(t,x,u) = h(t,x)\chi_\omega(x),$$

is given. Here $(t,x) \in (0,T) \times (0,1)$, $\omega = (\alpha, \beta) \subset\subset [0,1]$, and f is locally Lipschitz with respect to u.

Population dynamics with infinite time delay

Maurizio Grasselli

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Joint work with C. Cavaterra

A sufficiently general class of population dynamics models with spatial diffusion and infinite time delays is considered. The infinite delays are represented through time-dependent relaxation (or memory) kernels which are not necessarily monotone decreasing, even though they decay exponentially fast. The main goal of the talk is to show how the corresponding integro-partial differential system can be analyzed within the theory of infinite-dimensional dynamical systems in order to prove, e.g., the existence of global and exponential attractors (see [1]). In particular, when the kernels are rescaled by a relaxation time $\varepsilon > 0$, it can be demonstrated that the global dynamics of the system is sufficiently "close" to the one with no delay effects, provided that ε is small enough. If time permits, we shall also discuss the case in which the memory kernels depend on the spatial variables as well (see [2,3]).

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Convergence to a stationary state for solutions of semilinear parabolic inverse problems

Davide Guidetti

University of Bologna, Italy

Joint work with F. Colombo

We consider an inverse problem, with an abstract semilinear parabolic integrodifferential equation. The convolution kernel, together with the solution of the parabolic problem, is unknown. A supplementary condition, in the form of a certain functional applied to the (unknown) solution is given. The aim is to reconstruct the solution, together with the convolution kernel. This problem has been recently considered by many authors from many points of view. Our aim is to look for sufficient conditions, ensuring that the solution is globally defined and converges to a stationary state, as time t goes to $+\infty$. The main tools are maximal regularity results for abstract parabolic systems in an L^1-setting, together with some standard facts concerning boundedness of solutions of global linear parabolic abstract systems.

Optimal control of thermomechanical phase transitions

Dietmar Hoemberg

Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany

Joint work with K. Chelminski

In the talk I will discuss a thermomechanical model of phase transitions, where the quasistatic equations of thermoelasticity are coupled with a system of rate laws for the phase fractions of the different phases. In comparison to classical thermoelasticity the main effect stems from different thermal expansion coefficients of the respective phases, which are taken care of by a mixture ansatz. We will derive existence and stability results. The main part of the talk will be devoted to the corresponding control problem. A typical task is to derive the optimal cooling strategy in order to obtain a certain phase mixture and/or desired mechanical properties. We prove existence of optimal controls, derive first order optimality conditions and conclude with some numerical results.

Recovering memory kernels in parabolic transmission problems

Alfredo Lorenzi

University of Milan, Italy

We are concerned with recovering two unknown kernels related to a thermal body Ω with memory consisting of two different thermal bodies Ω_1 and Ω_2 with (in general) different memories. In other words, the problem under investigation is a transmission problem where the boundaries of the two thermal bodies Ω_1 and Ω_2 have a common (closed) surface Γ intersecting the boundary $\partial\Omega$ of Ω. This implies the the boundaries $\partial\Omega_1$ and $\partial\Omega_2$ are (in general) non-convex and endowed with a manifold consisting of corner points. Therefore the non-smoothness of $\partial\Omega_1$ and $\partial\Omega_2$ will oblige us to introduce local Sobolev L^2-spaces related to Ω_1 and Ω_2, when dealing with further regularity of the solution to the direct problem.

Surprisingly up to now the identification of unknown kernels in transmission problems seem not to be investigated.

To deal with a problem of this type, we need to describe the properties we require to the related open domains: *i)* Ω_1 and Ω_2 are n-dimensional open bounded domains; *ii)* $\Gamma = \partial\Omega_1 \cap \partial\Omega_2 \neq \varnothing$; *iii)* $m_{n-1}(\Gamma) > 0$; *iv)* $\Gamma,\ \partial\Omega_1,\ \partial\Omega_2 \in C^2$; *v)* $\Omega = \Omega_1 \cup \overset{0}{\Gamma} \cup \Omega_2$; *vi)* $\partial\Omega = \partial\Omega_1 \cup \partial\Omega_2$; *vii)* $\Gamma_j \subset\subset \partial\Omega_j \setminus \Gamma$ is an open subset in $\partial\Omega_j$, $j = 1,2$; *viii)* $m_{n-1}(\Gamma_j) > 0$, $j = 1,2$.

The *direct problem* related to ours (conventionally called an *inverse problem*) consists in searching for a pair of functions $u_j : \Omega_j \times (0,T) \to \mathbf{R}$, $j = 1,2$, such that

$$D_t u_j - \operatorname{div}(a_j \nabla u_j + a_j h_j * \nabla u_j) + b_j u_j = f_j, \quad \text{in } \Omega_j \times (0,T),\ j = 1,2, \tag{0.4}$$

$$u_j(x,0) = u_{j,0}(x), \quad x \in \Omega_j,\ j = 1,2, \tag{0.5}$$

$$a_1 D_\nu u_1 + a_1 h_1 * D_\nu u_1 = a_2 D_\nu u_2 + a_2 h_2 * D_\nu u_2, \quad \text{on } \overset{0}{\Gamma} \times (0,T)\,, \tag{0.6}$$

$$u_1 = u_2, \quad \text{on } \overset{0}{\Gamma} \times (0,T)\,, \tag{0.7}$$

$$u_j = 0, \quad \text{on } (\partial\Omega_j \setminus \overline{\Gamma}) \times (0,T),\ j = 1,2, \tag{0.8}$$

where ν stands for the outer normal unit vector on $\partial\Omega_2$ and $h * v(t) = \int_0^t h(t-s)v(s)\,ds$. The identification problem related to (0.4)-(0.8) consists in recovering, in addition to the pair (u_1,u_2), also the pair $(h_1,h_2) \in L^2((0,T)) \times L^2((0,T))$, whenever a couple of supplementary information is given. In our case they are the following flux-type conditions:

$$\Phi_j[u_j(\cdot,t) + h_j u_j(\cdot,t)] = g_j(t), \quad \text{for a.e. } t \in (0,T),\ j = 1,2, \tag{0.9}$$

where

$$\Phi_j[z] = \int_{\Gamma_j} \varphi_j(x) a_j(x) D_\nu z(x)\,d\sigma(x), \quad j = 1,2, \tag{0.10}$$

σ denoting the (Lebesgue) surface measure.

Under the assumptions that the coefficients a_j, b_j and the data f_j, $u_{j,0}$, $j = 1,2$, should be smooth enough and satisfy *suitable solvability conditions*, existence and uniqueness

results are proved for (h_1, h_2), provided the state functions $u_1(t,\cdot)$ and $u_2(t,\cdot)$, $t \in [0,T]$, take their values in $L^2(\Omega_1)$ and $L^2(\Omega_2)$, respectively.
We stress that, to recover (h_1, h_2), we need that only *small parts* Γ_1 and Γ_2 of Ω_1 and Ω_2 are accessible to perform the needed measurements (0.9).

On Ingham type theorems and their applications

PAOLA LORETI

University of Roma La Sapienza, Italy

Let $(\lambda_k)_{k\in K}$ be a family of real numbers, and consider the functions of the form

$$f(t) = \sum_{k\in K} a_k e^{i\lambda_k t}$$

with complex coefficients (a_k). We recall a classical theorem due to Ingham (1936):

Theorem. Assume that

$$\gamma := \inf_{j\neq k} |\lambda_j - \lambda_k| > 0,$$

and let I be a bounded interval of length $|I| > 2\pi/\gamma$. There exist two constants $c_1, c_2 > 0$ such that

$$c_1 \sum_{k\in K} |a_k|^2 \leq \int_I |f(t)|^2 \, dt \leq c_2 \sum_{k\in K} |a_k|^2$$

for all square summable sequences (a_k) of complex numbers.

We discuss the proof, some generalizations and applications.

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On boundary stabilizability of hyperbolic conservation laws

ANDREA MARSON

University of Padova

We start by considering the problem of stabilizing around an equilibrium a gas in a cylinder with a moving piston. The system in modeled by isentropic gas dynamics in lagrangian coordinates in one space dimension. What comes out is a boundary value problem for a system of conservation laws in the interval $[0, 1]$, with a control acting at the boundary $x = 1$ on a subset of the conserved quantities. We first study the well posedness of the boundary value problem and then write conditions on the boundary data so that it is possible to find out a control that stabilizes the system. The paper is written in collaboration with F. Ancona from the University of Bologna.

Approximation of solutions to linear and nonlinear integro-differential parabolic equations in L^p-spaces

Francesca Messina

University of Milano

Joint work with A. Lorenzi

This talk concerns two paper: in the first we show that, under suitable assumptions, the solutions to the following approximating initial and boundary value problems P_ε, defined below, converge in $L^p((0,T); L^s(\Omega))$, for suitable indexes $p \in [1, +\infty)$ and $s \in [1, +\infty)$, to the solution to the *limit problem* P_0. Consequently, we will consider the initial value problem depending on the (small) positive parameter ε in a suitable $L^s(\Omega)$-space, where $\Omega \subset \mathbf{R}^n$ is a bounded domain with a C^2-boundary, for every $t \in (0, T)$:

$$(P_\varepsilon) \quad \begin{cases} u'_\varepsilon(t) - \chi(\varepsilon)Au(t) - A((k_\varepsilon + \varphi(\varepsilon)h_\varepsilon) * u_\varepsilon)(t) = f_\varepsilon(t), \\ u_\varepsilon(0) = u_{0,\varepsilon}. \end{cases}$$

In problem (P_ε) we use the following notation:

$$l_\varepsilon(t) = \varepsilon^{-1} l(t/\varepsilon), \quad l_\varepsilon * u_\varepsilon(t) = \int_0^t l_\varepsilon(t-s)u_\varepsilon(s)\,ds.$$

The *limit problem* (P_0) is

$$(P_0) \quad \begin{cases} u'(t) - (1+\chi_0)Au(t) = f(t), & \forall t \in (0, T], \\ u(0) = u_0. \end{cases}$$

We will assume A is a linear unbounded operator and χ and φ are *positive scalar* functions, defined on $(0, \varepsilon_0]$, such that

$$\chi(\varepsilon) \to \chi_0 \geq 0 \quad \text{and} \quad \varphi(\varepsilon) \to 0 \text{ as } \varepsilon \to 0+,$$

In particular we consider the following class of *explicit* second-order differential operators A in divergence form:

$$A := A(x, D_x) = -\sum_{i,j=1}^{n} D_{x_i}[a_{i,j}(x)D_{x_j}] + a_0(x),$$

endowed with vanishing Dirichlet boundary conditions.

In the second paper we show that, under suitable assumptions, the solutions to the approximating nonlinear problem (P'_ε), defined below and related to a second-order differential operator A uniformly elliptic, converge in $L^p((0,T);L^\alpha(\Omega))$–norm, for suitable $p \in [1,+\infty)$ and $\alpha \in [1,+\infty)$, to the solution to the *limit problem* (P'_0):

$$(P'_\varepsilon) \begin{cases} u'_\varepsilon(t) - \chi(\varepsilon)Au(t) - A((k_\varepsilon + \varphi(\varepsilon)h_\varepsilon) * u_\varepsilon)(t) = f_\varepsilon(t) + Nu_\varepsilon(t), & \forall t \in (0,T], \\ u_\varepsilon(0) = u_{0,\varepsilon}, \end{cases}$$

$$(P'_0) \quad \begin{cases} u'(t) - (1+\chi_0)Au(t) = f(t) + Nu(t), & \forall t \in (0,T], \\ u(0) = u_0. \end{cases}$$

In particular, we consider two different nonlinearities, which are locally Lipschitz-continuous in suitable norms: N admits either of the following representations

(B)

$$N(u)(t,x) = \int_\Omega l(t,x,y)\Psi(u)(y)dy, \quad u \in L^\alpha(\Omega),$$

where the kernel l is a smooth function from $[0,T]\times\Omega\times\Omega$ into $\mathbf{R}$ and Ψ is a locally Lipschitz nonlinear operator from $L^\alpha(\Omega)$ into $L^{\alpha_0}(\Omega)$, with $\alpha > \alpha_0$;

(C)

$$Nu(x) = \psi(u(x)),$$

where ψ is a smooth function from $\mathbf{R}$ into $\mathbf{R}$.

The results proved in the linear case are used here as basic tools to solve our problems. In the papers, fundamental existence and approximation results are proved - in a more general Banach-space framework - involving a generalization of the kernels k, h and of the operators A, N. Such results are needed to simplify the proofs of our theorems in the case of the *explicit* differential operator A.

Some generalizations of the Cahn-Hilliard equation

Alain Miranville

University of Poitiers, France

Our aim in this talk is to present and discuss models of Cahn-Hilliard equations for phase separation in binary alloys. These models are derived based essentially on a balance law for internal microforces proposed by M. Gurtin. This microforce balance takes into account the interactions at a microscopic level, whereas standard forces are associated with macroscopic length scales (the difference of length scales may justify the need for a separate balance law for microforces; one thus obtains two-scale models). One advantadge of this derivation is that it allows to take into account effects such as anisotropy, mechanical effects and thermal effects (we can note that it is not clear, in the classical derivation, how to take into account such effects). One can also extend this approach to multicomponent alloys.

Input identification to a class of nonlinear input-output causal systems

Luciano Pandolfi

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Joint work with F. Ferri

In this paper we consider an input-output process described by the following nonlinear integral equation:

$$y(t) = \int_0^t K(t,s)u(s)ds + \int_0^t F(t,s,u(s))ds, \quad t \in [0,T]\,.$$

We want to reconstruct the signal u from measures taken on the output y.

We combine a monotonicity assumption and the extraction of the "dominant" part $k(t,t)$ so to have a reconstruction in square mean and also in the uniform norm on those intervals on which the unknown input u is continuous.

For practical applications to control theory (noise suppression, signal tracking...) the reconstruction algorithm must be causal. For this reason we use Lavrent'ev method: we introduce the equation

$$\epsilon v(t) + \int_0^t K(t,s)v(s)ds + \int_0^t F(t,s,v(s))ds = y(t)\,, \qquad 0 \le t \le T$$

and we give conditions under which the solution $v(t)$ exists on $[0,T]$ and

$$\lim_{\epsilon \to 0+} v(t) = u(t)$$

both in $L^2(0,T)$ and in $C(a,b)$ if it happens that the unknown input u is regular on $[a,b]$.

We observe that we don't use monotonicity of the integral operators in the usual L^2 norms and that a special case, the case $F(t,s,u) = F(t,s)\Phi(u)$ (under suitable assumptions) could be handled using the linear theory: just reconstruct $u+\Phi(u)$ with the methods of the linear theory and then solve for u. It is noteworthy that the algorithm we present give directly an approximation of u without the need of explicit inversion.

The key assumptions are

- the unknown input u is an n-vector valued function which is piecewise of class $W^{1,2}$ (so that K is an $n \times n$ matrix);
- for each $t \in [0,T]$ we have $K(t,t) > 0$;
- the transformation from R^n to itself $u \to F(t,t,u)$ is monotonic for each $t \in [0,T]$.

Monotonicity of the integral operators in the original L^2 topology is not required.

Control of fluxes on networks

Benedetto Piccoli

Istituto per le Applicazioni del Calcolo M. Picone (IAC), CNR, Roma, Italy

We consider the problem of optimizing traffic distribution coefficients for a fluidodynamical model of traffic on networks.

Observability estimates for heterogeneous Maxwell's system

Cristina Pignotti

University of Aquila, Italy

Joint work with S. Nicaise

The exact boundary or internal controllability and stabilization of Maxwell's equations with constant coefficients have been studied by many authors and are usually based on an observability estimate obtained by different methods like the multiplier method, microlocal analysis, the frequency domain method.

More recently, several papers are devoted to the stability and/or controllability of Maxwell's system with variable coefficients. All of them are based on an observability estimate which is obtained using the multiplier method but leading to a relatively strong assumption on the permettivity and permeability coefficients.

In the case of wave equations with non constant leading coefficients (or more complex scalar systems, like Petrowski system or Schrödinger equations), a more powerful tool combining differential geometry, Carleman's estimates and microlocal analysis was recently developed by Lasiecka, Triggiani and Yao, to obtain observability estimates under quite general assumptions, related to the geometric properties of the domain.

Since Maxwell's system is reducable to a perturbed vectorial wave equation but with a decoupled principal part, the above tool is applicable as well. In this talk we will present some recent results obtained applying this technique to Maxwell's system under the geometric assumption of the related wave equation. First, we will give a Carleman's estimate for Maxwell's system. Then, we will apply this estimate obtaining boundary and internal observability estimates. We will also give some examples, where the geometric assumption is illustrated.

Spectral decomposition and asymptotic stability for a class of aeroacustic models

J.-Pierre Raymond

University Paul Sabatier, Toulouse, France

Joint work with L. Cot and J. Vancostenoble

We consider systems of coupled partial differential equations modelling the interaction between the propagation of acoustic waves in a bounded domain and a structure located in a part of the boundary of the domain. Different kind of structures are considered: membranes, plates, and structures with rigid oscillations. All these models may be written in an abstract framework. Due to the coupling terms, this kind of models cannot be written in the form of a second order evolution equation involving a self-adjoint operator. For this reason, the existence of a basis allowing a spectral decomposition of solutions is not obvious. Using a symmetrization procedure, we prove the existence of a suitable basis for which a spectral decomposition of solutions may be performed.

As an application, we consider the problem of the asymptotic behavior of solutions for such models when they are subjected to a nonlinear weak damping. Using this suitable basis, we give a necessary and sufficient condition insuring the strong asymptotic stability of the abstract model under standard assumptions on the feedback. As a consequence, in applications, the problem of asymptotic stability reduces to the proof of a unique continuation result. We shall illustrate this result for various aeroacoustic models and different types of feedbacks.