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Geography of order and chaos in mechanics. Investigations of quasi-integrable systems with analytical, numerical, and graphical tools. (English)

As announced in the book's subtitle itself, the author carries out "investigations of quasi-integrable systems with analytical, numerical and graphical tools", The book addresses mainly the undergraduate and graduate student audience, but should also be of interest for researchers in physics willing to deepen their knowledge about the mathematical foundations of classical mechanics.

The book proposes an exciting exploration of dynamical systems from celestial mechanics, providing the reader with all the necessary theoretical and numerical tools, for a good understanding of the problems themselves and the resolution techniques. After an introductory survey, giving a good overview of the content and the methodology of the book, the different topics are divided into 8 chapters, corresponding to three main subjects: theoretical tools, numerical tools and study of concrete examples.

The first part of the book (Chapters 2 and 3) concerns theoretical tools. Chapter 2 starts with a crash-course in differential and symplectic geometry. This exposition is very concise, almost without examples, but presents exclusively concepts and objects which have some relevance for the rest of the book. First, the author recalls basic definitions (manifolds, differentiable maps, tensor fields, Lie derivatives, etc.) and then basic constructions of symplectic geometry (symplectic forms, canonical transformations, Darboux theorem, cotangent bundle, Poisson brackets, etc.). The author carries on with a few relevant facts in the theory of Lie groups and Lie algebras (adjoint and coadjoint representation, actions of Lie groups on manifolds, some facts about $SO(2)$ and $SO(3)$).

After these basics, the second half of Chapter 2 is devoted to Lagrangian and Hamiltonian mechanics, and in particular to completely integrable Hamiltonian systems. After defining the Hamiltonian and Lagrangian formulation of classical mechanics, the author discusses the problem of taking advantage of symmetries in a given mechanical system in order to reduce it to a lower-dimensional system. Finally, the definition of Liouville complete integrability is given, and this introductory Chapter 2 ends with the formulation and the proof of Arnold's theorem about the existence (away from singularities) of a fibration in invariant Lagrangian tori and of associated action-angle coordinates.

In Chapter 3, the author discusses perturbation theory of completely integrable Hamiltonian systems, namely Hamiltonians of the form $H_\epsilon = H_0 + \epsilon H_1$, with H_0 a completely integrable Hamiltonian and $\epsilon \ll 1$. Such a system is called "quasi-integrable". Rather than a typical "first theorem, then proof" exposition, the books follows an explorative exposition, and drives the reader through the growing difficulties arising when one

tackles this perturbative problem. Namely, one tries to find a suitable canonical transformation depending on ϵ which brings the perturbed Hamiltonian H_ϵ back to a "nice" form by means of an expansion in the parameter ϵ. The first step is to consider the formal level of this expansion, i.e., without taking care of convergence problems. The author explains how the use of angle-action coordinates and Fourier series expansion in the angle variables yields to the so-called "homological equation". The author discusses the "small denominators problem" arising when trying to solve this equation and explains how certain "Diophantine conditions" solve the apparently hopeless problem. Equipped with these tools, the author gives precise insight into the proofs of two versions of the celebrated K.A.M theorem (according to Arnold and to Kolmogorov), as well as Nekhoroshev's theorem. These very important theorems predict how the "geography" of the phase space changes from the completely integrable system H_0 to the perturbed one $H_0 + \epsilon H_1$: persistence of particular (strongly non-resonant) tori, destruction of resonant tori, exponentially long stability, manifestation of chaos.

The second part of the book (Chapters 4 and 5) concerns numerical methods suitable for studying problems from celestial mechanics, which are implemented in several MATLAB programs provided with the book. The author gives in Chapter 4 the rudiments of the numerical integration of ordinary differential equations. He describes various general methods (Euler, Runge-Kutta, etc.) as well as geometric methods, which preserve some geometric properties of the ODE system under consideration, namely, symplectic methods. In Chapter 5, another sort of numerical tools is presented, namely methods based on frequency analysis by means of Fourier transform which are particularily well adapted to quasi-integrable systems. The book explains several refinements of the Fourier transform method: fast Fourier transform (FFT), frequency modified Fourier transform (FMFT), wavelets and time-frequency analysis, frequency modulation indicator (FMI).

After reading the first 5 chapters, the reader is very well equipped to study theoretically and numerically problems from celestial mechanics, which is the subject of the last 4 chapters of the book. The author considers various problems: the Kepler problem, the perturbed Kepler problem, the circular restricted three-body problem, the 3-dimensional planetary problem. For each of these problems, the author provides a rich analysis composed of theoretical considerations, investigation of the symmetries and corresponding reduction to simpler systems, and numerical illustrations/verifications of the results with the help of the provided MATLAB programs.

In my opinion, the announced goal of the book is reached: "understand the qualitative and quantitative features of the relative dynamics, even for systems with three or more degrees of freedom [...], grasp the geography of the resonances, and hence the distribution of order and chaos."

This book provides the reader with many illustrations and graphics from numerical simulations. It is concise and well written and constitutes a self-contained introduction to the subject for individual readers, but provides also good material for preparing lectures on celestial mechanics.

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Keywords : Hamiltonian mechanics; integrable systems; perturbations; nearly integrable systems; numerical simulations; celestial mechanics; KAM theorem; Kepler problem;
`MATLAB`

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Classification :

   ∗70-01 Textbooks (mechanics of particles and systems)
   37J40 Perturbations, etc.
   70F15 Celestial mechanics
   37J05 Relations with symplectic geometry and topology
   70H08 Nearly integrable Hamiltonian systems, KAM theory
   70H33 Symmetries
   65P10 Hamiltonian systems including symplectic integrators
   37J15 Symmetries, etc.
   37N05 Dynamical systems in classical and celestial mechanics
   37M10 Time series analysis
   65L06 Multistep, Runge-Kutta, and extrapolation methods