Lectures

1-2 Definition of Riemann Surface (RS). Examples: $\mathbb{C}$, open subsets and topological covers of a RS, $\mathbb{C}_\infty$ (one point compactification of $\mathbb{C}$), S^2, $\mathbb{P}^1(\mathbb{C})$. [Fo] I.1, [M] I.1,2, [D] 3.1, 3.2.1.

3-4 Examples: Complex torus, graph of a holomorphic function on a domain, zero locus of holomorphic maps $\mathbb{C}^2 \to \mathbb{C}$, smooth affine curve. [Fo] I.1, [M] I.2 [D] 3.2.2.

5-6 Holomorphic maps between RS, theorem on the local behaviour of a non-constant holomorphic map. Corollaries: Thm. of the open map, hol. + inj. implies bihol. [Fo] I.1,2, [M] II.1,3, [D] Prop.5.

7-8 Ramification locus and branch locus, holomorphic non constant maps between RS have discrete fibers, the degree of a non-constant holomorphic map between compact RS (Statement in the case of proper holomorphic maps between RS) [Fo] I.4, [M] II.4, [D] Prop.6, Prop.7.

9-10 RS as orientable differential 2-manifolds. Riemann Hurwitz Formula and corollaries. Meromorphic functions as holomorphic maps to P^1 and corollaries. [Fo] I.4, [M] II.4, II.1.

11-12 Meromorphic functions on $\mathbb{P}^1$: map induced by a polynomial, $\mathbb{C}[z]$ and $\mathbb{C}(z)$. The Fundamental Theorem of Algebra, explicit computations, the automorphism group of $\mathbb{P}^1$. [Fo] I.1, [M] II.2, II.3.

13-14 Construction of RS by glueing. Riemann's Existence Theorem, relation between theory of topological coverings and theory of holomorphic maps between compact RS. [M] III.1, III.4, [Fu] 18a, [D] 4.2.2.

15-16 Hyperelliptic curves explicit constructions as application of Riemann's Existence Theorem, via the corresponding monodromy homomorphism, and by glueing affine plane curves. [Fo] I.5, [M] III.1, III.4.

17-18 Application of Riemann's Existence Theorem: compactification of a plane affine curve using the projection to the first coordinate. Projective smooth plane curves and projection from a point outside a projective plane curves (example: Fermat). Application of Riemann's Existence Theorem: the compact RS associated to a homogenous irreducible polynomial in 3 variables. [D] 3.2.2, 4.2.3, [M] I.2.3.

19-20-21 Holomorphic maps between complex tori, the space parametrizing complex tori up to biholomorphism. [M] III.1, [S] Ch.1.

22 Divisors, principal divisors, the degree of a divisor, the Picard group. [M] V.1, [Fo] 16.1-16.3.

23-24 Picard groups, linear equivalence of divisors ([M] V.3). The Riemann Roch space $L(D)$, finite dimensionality, examples for $X = \mathbb{P}^1$,

25-26 Canonical divisors ([M], V, def. 18), example for $\mathbb{P}^1$, the canonical (divisor) class. The Riemann-Roch theorem (with Serre duality) and corollaries ([M] VI.3, [Fo] 16,17, [Fu] 21c).

27-28 The topological and the analytical genus are the same (cf. [Fu], Prop. 20.14). Corollaries of Riemann-Roch for R.S. of genus 0,1,2.

29-30 Sketch of the proof of the Riemann-Roch theorem: sheaves ([Fo] I.6, [M] IX), finite dimensionality of cohomology groups ([Fo] 14, p.109] and Serre duality ([Fo] Thm 17.11, [M] Thm VI 3.3). The adjunction formula and the genus of plane curves of degree d.

31-32 Holomorphic differential forms on affine algebraic curves. Holomorphic differential forms on hyperelliptic and plane algebraic curves ([M] Exercises p.111-112).

33-34 Maps to projective space ([Fo] 17.21, [M] Lemma V 4.2), hyperplane sections and linear systems ([M] p147), very ample divisors ([M] p.163), a criterium for very ampleness ([M] p161-163).

35-36 Examples of maps to projective space: $\mathbb{P}^1$, genus 1 R.S. and cubic curves, polynomial equations for the image of a R.S. in projective space.

37-38 The canonical map, the hyperelliptic case, examples in genus 3,4,5 ([M] VII.2).

39-40 The geometric form of Riemann-Roch ([M] VII.2), trigonal curves. The Picard group and the addition law on a cubic curve.

41-42 Final remarks (not for the exam) on the Jacobian and the theta divisor. ([Fo] 20, 21, [M] VIII, [Fu] 21d).

Bibliography:

[D] S. Donaldson, Riemann Surfaces, Oxford GTM, 22, Oxford Mathematics
[Fo] O.Forster, Lectures on Riemann Surfaces, GTM 81, Springer
[Fu] W.Fulton, Algebraic Topology A First Course, GTM 153, Springer
[M] R.Miranda, Algebraic Curves and Riemann Surfaces, GSM 5, AMS
[S] J. Silverman, Advanced topics in the arithmetic of elliptic curves, GTM 151. Springer-Verlag 1994.