Lectures

1-2 Complex analysis in 1 variable, definition of Riemann Surface (RS). Examples: $\mathbb{C}_\infty$ (one point compactification of $\mathbb{C}$), S^2. [F] I.1, [M] I.1,2.

3-4 Examples: $P^1(\mathbb{C})$, complex torus, graph of a holomorphic function on a domain, plane smooth affine curve, plane smooth projective curve. [F] I.1, [M] I.2,3.

5-6 Holomorphic functions on a RS, holomorphic maps between RS, theorem on the local behaviour of a non-constant holomorphic map.

Corollaries: Thm. of the open map, hol. + inj. implies bihol, hol. map from P^1 to P^1 defined by a polynomial, Fundamental Theorem of Algebra. [F] I.1,2, [M] II.1,3.

7-8 Ramification locus and branch locus, holomorphic maps between RS have discrete fibers, the degree of a non-constant holomorphic map between compact RS. Example: map induced by a polynomial. RS are orientable. The classification theorem of compact topological surfaces, Euler characteristic. [F] I.4, [M] II.4.

9-10 The Riemann Hurwitz formula and corollaries. Example: projection to the first coordinate for an affine plane curve, projection from a point outside a projective plane curves (example: Fermat). [F] I.4, [M] II.4.

11-12 Meromorphic functions, meromorphic functions as holomorphic maps to P^1 and corollaries, meromorphic functions on P^1, the automorphism group of P^1. [F] I.1, [M] II.1,2,3.

13-14 Holomorphic maps between complex tori, the space parametrizing complex tori up to biholomorphism. [M] III.1.

15-16 Construction of hyperelliptic curves by glueing affine plane curves, Riemann's Existence Theorem. [F] I.5, [M] III.1,4.

17-18 Consequences of Riemann's Existence Theorem: relation between theory of coverings and theory of holomorphic maps between compact RS. The De Rham cohomology of a compact oriented differentiable surface X. Construction of a special basis of $H^1_{dR}(X)$. [M] III.4, [Fu] 18a.

19-20 Integration of closed 1-forms along a smooth path on a differentiable surface and its homotopic properties. The De Rham theorem for a compact Riemann surface: $H^1_{dR}(X) \simeq Hom_{\mathbb{Z}}(H_1(X), \mathbb{R})$. Poincaré Duality and Poincaré duals using the special basis. [Fu] 1a,b,9b,18a,b,c.

21-22 Differentiable 1-forms, (1,0) and (0,1)-forms and holomorphic 1-forms on a RS. [Fu].

Divisors, principal divisors, the degree of a divisor, the Picard group. [M] V.1, [Fo] 16.1-16.3.

23-24 The Riemann Roch space $L(D)$, finite dimensionality, Examples for $X = P^1$, Picard groups.

25-26 Linear equivalence of divisors ([M] V.3). Canonical divisors ([M], V, def. 18), example for P^1, the canonical (divisor) class. The Riemann-Roch theorem (with Serre duality) and corollaries ([M] VI.3, [Fo] 16,17, [Fu] 21c).

27-28 Riemann-Roch for R.S. of genus 0,1,2. The topological and the analytical genus are the same (cf. [Fu], Prop. 20.14). Holomorphic differential forms on affine algebraic curves ([M] Exercises p.111-112).

29-30 Holomorphic differential forms on hyperelliptic, Fermat and plane algebraic curves ([M] Exercises p.111-112).

31-32 Sketch of the proof of the Riemann-Roch theorem: sheaves ([Fo] I.6, [M] IX), stalks of sheaves, homomorphisms of sheaves ([Fo] 15, [M] IX.2), exact sequences of sheaves ([Fo] 15.5), long exact cohomology sequence, finite dimensionality of cohomology groups ([Fo] 14, p.109] and Serre duality ([Fo] Thm 17.11, [M] Thm VI 3.3).

33-34 Maps to projective space ([Fo] 17.21, [M] Lemma V 4.2), very ample divisors ([M] p.163), hyperplane sections and linear systems ([M] p147), criterium for very ampleness ([M] p161-163).

35-36 Examples of maps to projective space: genus 1 R.S. and cubic curves, polynomial equations for an embedded R.S.

37-38 The canonical map, the hyperelliptic case, examples in genus 3,4,5 ([M] VII.2).

39-40 The geometric form of Riemann-Roch ([M] VII.2), trigonal curves. The Picard group and the addition law on a cubic curve.

41-42 Final remarks (not for the exam) on the Jacobian and the theta divisor. ([Fo] 20, 21, [M] VIII, [Fu] 21d).

Bibliography:

[F] O.Forster, Lectures on Riemann Surfaces, GTM 81, Springer
[M] R.Miranda, Algebraic Curves and Riemann Surfaces, GSM 5, AMS
[Fu] W.Fulton, Algebraic Topology A First Course, GTM 153, Springer