Analisi Matematica 1- Corso di Laurea in Fisica

ESERCIZI –Foglio 1

1. Risolvere le seguenti disequazioni:

   (a) $x^3 + 8 < 0$;

   (b) $\sqrt{x+8} < 12 - x$

   (c) $\sqrt{x+1} \leq 5 - \sqrt{x+6}$

   (d) $\dfrac{\sqrt{x - \sqrt{1-x}}}{1-\sqrt{x}} < 1$

   (e) $2^{x+1} > 4 \cdot 2^{-\sqrt{2|x|}}$

   (f) $\left(\dfrac{1}{2}\right)^{(x-\sqrt{4-x^2})} \geq 1$

   (g) $\cos x + \sin x \leq 0, \qquad x \in [0, 2\pi)$

   (h) $x - 1 + (3x-1)\log x \leq 0.$

   (i) $\dfrac{1}{2} < \sqrt{\dfrac{x-a}{x}} < 1, \qquad$ al variare del parametro $a \in \mathbb{R}$

2. Determinare **estremo superiore, estremo inferiore** e, qualora esistano, **massimo** e **minimo** dei seguenti sottoinsiemi di $\mathbb{R}$.

   (a) $E = \left\{ x \in \mathbb{R} \ : \ x = 3^n - \dfrac{1}{n^2}, \quad n \in \mathbb{N} \right\}$

   (b) $E = \{x \in \mathbb{R} \ : \ -1 \leq x < 1\} \cup \{15\}$

   (c) $E = \left\{ x \in \mathbb{R} \ : \ x = \dfrac{2n^2 - 1}{n^2}, \quad n \in \mathbb{N} \right\}$

(d) $E = \{z \in \mathbb{R} \,:\, z = x\,y, \quad x, y \in \mathbb{R},\ -2 \le x \le 1,\ -1 \le y < 0\}$

(e) $E = \left\{ x \in \mathbb{R} \,:\, \sin x > -\dfrac{\sqrt{3}}{2}, \quad x \in [-\pi, \pi) \right\}$

(f) $E = \left\{ x \in \mathbb{R} \,:\, \dfrac{\sqrt{x^2 - 5x + 6}}{x - 2} \le 1 \right\}$

(g) $E = \{\alpha \in \mathbb{R} \,:\, \alpha = (x+1)\,|2y - 3|\,,\ -1 < x < 2,\ 1 < y \le 4\}$

(h) $E = \{x \in \mathbb{R} \,:\, 0 < x < 3\} \cup \left\{ x \in \mathbb{R} \,:\, x = 4 - \dfrac{1}{n^2}, \quad n \in \mathbb{N} \right\}$

(i) $E = \left\{ x \in \mathbb{R} \,:\, x = \cos(\pi n) + \dfrac{2+n}{n+1}, \quad n \in \mathbb{N} \right\}$