Analisi Matematica 1- Corso di Laurea in Fisica

ESERCIZI –Foglio 6

1. Calcolare, se esiste, $\lim_{n\to+\infty} a_n$:

   (a) $a_n = \dfrac{\log n - 3^n(\log n)^4}{\cos(n^n) + (n\log n)^3 + 2^n}$; (b) $a_n = (n-\sqrt{n})\left(\sqrt[3]{1+\dfrac{2}{n}}-1\right)$;

   (c) $a_n = \log(3^n+n)\log\left(\dfrac{n^2+n}{n^2-3n}\right)$; (d) $a_n = n^2\left(e^{\frac{1}{n^2}} - \cos\dfrac{1}{n}\right)$;

   (e) $a_n = \sqrt[n]{n\log n}$; (f) $a_n = \dfrac{n^3+2n}{5-n}\log\left(\cos\left(\dfrac{3}{n^2}\right)\right)$.

2. La successione
$$x_n = \frac{(5n)^n - 50^n - n^4 e^{3n}}{n e^{2n} + n^{n+5\log n} + 3^n}$$
   tende a ....

3. Stabilire se sono vere o false le seguenti relazioni per $n \to +\infty$:

   (a) $\sin(n^2) \sim \frac{1}{n^2}$;

   (b) $\sin\frac{n^2+1}{n^4+2n} \sim \tan\frac{1}{n^2+1}$

   (c) $e^{n^2+\sqrt{n}} \sim e^{n^2+3n}$

   (d) $\left(1+\frac{2}{n+1}\right)^{\frac{1}{3}} \sim \sqrt{\log(1+\frac{4}{n^2})}$

   (e) $(n+1)! \sim n\, n!$.

4. Calcolare, se esiste, $\lim_{n\to+\infty} a_n$:

   (a) $a_n = \dfrac{n\sin\left(\frac{1}{\sqrt{n}}\right)}{\arctan((-1)^n n) + 4\log n}$; (b) $a_n = n\cos\left(e^{\frac{\sqrt{n+2}}{n+1}}\right)$;

   (c) $a_n = (n+1)\left(e^{\frac{n+1}{n+2}} - e\right)$ (d) $a_n = (\sqrt[n]{2}-1)^n$;

   (e) $a_n = \dfrac{e^{\sqrt{(\ln n)^2+\ln n^4}}}{n^2+1}$; (f) $a_n = (n^3+2)\log\left(1+\dfrac{5n}{n^4+2}\right)$;

   (g) $a_n = n^2\left(\log(3+n^2) - \log(2+n^2)\right)$; (h) $a_n = \dfrac{\tan\left(\frac{\pi}{2}+\frac{1}{n^2}\right)}{n^3\left(e^{2/n}-1\right)}$;

   (i) $a_n = n^3\left(\tan\dfrac{3}{n} - \sin\dfrac{3}{n}\right)$; (l) $a_n = (1+n^2)^{\frac{2}{\log n}}$.

5. Calcolare il limite $\lim_{n\to+\infty}(a_n)^{b_n}$ dove

$$a_n = 1 - \sin\left(\frac{\sqrt[3]{8n+1}}{n^2+1}\right), \quad b_n = n\sqrt[3]{n^2}.$$

6. Al variare del parametro $\alpha \in \mathbb{R}$ calcolare, se esiste, il $\lim\limits_{n\to+\infty} a_n$

(a) $a_n = n^\alpha \left(\cos \frac{\pi}{n^2+1} - 1\right)$ (b) $a_n = n^\alpha \sin \frac{1}{\log n}$

(c) $a_n = n^\alpha \left(1 - \left(1 - \frac{2}{n}\right)^4\right)$ (d) $a_n = n^\alpha \left(e^{1+\frac{1}{n}} - e\right)$

(e) $a_n = n^\alpha \left(\sqrt[5]{n^2+1} - \sqrt[5]{n^2}\right)$ (f) $a_n = n^\alpha \left(\log\left(\cos \frac{2}{n^2}\right) + \sqrt[5]{1+\frac{2}{n^4}} - 1\right)$

7. Calcolare, se esiste

$$\lim_{n\to+\infty}\left(\sqrt[n]{3} + \arctan\frac{2}{n}\right)^n; \qquad \lim_{n\to+\infty}\frac{2n}{\log(n^3)}\left(\log^2(n+1) - \log^2 n\right);$$

$$\lim_{n\to+\infty}\frac{1+\cos\left(\pi\sqrt{9+1/n^2}\right)}{\log^2\left(\cos\frac{1}{n}\right)}; \qquad \lim_{n\to+\infty}\frac{\log\left(\frac{n+1}{n+3}\right)}{2^{1/n} - \cos\left(\frac{1}{\sqrt{n}}\right)}.$$

8. Vero o falso?

a) $\sqrt[5]{1+\frac{1}{n^2}} - 1 = o(\tan\frac{1}{n})$;

b) $\exp\left(\sin\frac{1}{n}\right) = 1 + \frac{1}{n} + o\left(\frac{1}{n^2}\right)$;

c) $\sinh\frac{1}{n} = o(\frac{\log n}{n^2})$;

d) $\mathrm{e}^{n-5\log n} = o(\mathrm{e}^n)$;

e) $\mathrm{e}^n n! = o(n^{\frac{n}{2}})$.

9. Al variare del parametro $\alpha \in \mathbb{R}$, calcolare, se esiste, $\lim_{n\to+\infty} a_n$:

(a) $a_n = \sqrt[n]{\alpha^{2n}+1}$;

(b) $a_n = \dfrac{n^\alpha - \log n}{n^3+1}$;

(b) $a_n = \alpha^n \dfrac{(-3)^n \log(2^n+1) - 2^n n^6}{3^{n/2}\log(n^2+1) + 5n}$.