Analisi Matematica 1- Corso di Laurea in Fisica

ESERCIZI –Foglio 7

1. Determinare per quali valori di α e β (reali) è vera, per $n \to +\infty$, la seguente affermazione:

$\sqrt{\cos \frac{1}{n}} - \exp\left(\sin \frac{1}{n^2}\right) \sim \alpha n^\beta.$

2. Vero o falso?

a) $\sqrt[5]{1+\frac{1}{n^2}} - 1 = o(\tan \frac{1}{n})$;

b) $\exp\left(\sin \frac{1}{n}\right) = 1 + \frac{1}{n} + o\left(\frac{1}{n}\right)$;

c) $\sin \frac{1}{n} = o(\frac{\log n}{n^2})$;

d) $\mathrm{e}^{n-5\log n} = o(\mathrm{e}^n)$;

e) $\mathrm{e}^{1/(n^2+1)} - 1 = o(n^{-\frac{3}{2}})$;

f) $\sin(n^2) \sim \frac{1}{n^2}$;

g) $\sin \frac{n^2+1}{n^4+2n} \sim \tan \frac{1}{n^2+1}$;

h) $\mathrm{e}^{n^2+\sqrt{n}} \sim \mathrm{e}^{n^2+3n}$

i) $\left(1+\frac{2}{n+1}\right)^{\frac{1}{3}} \sim \sqrt{\log(1+\frac{4}{n^2})}$;

l) $(n+1)! \sim n\, n!$

3. Calcolare, se esiste, $\lim_{n\to+\infty} a_n$ dove:

(a) $a_n = n^3 \left(\sinh \frac{1}{n} - \sin \frac{1}{n}\right)$;

(b) $a_n = n^3 \left(\arctan \frac{2}{n} - \mathrm{e}^{\frac{2}{n}} + 1\right)$;

(c) $a_n = n^2 \left(\log\left(1 + \frac{1}{n} - \frac{2}{n^2}\right) - \frac{1}{n}\right)$;

(d) $a_n = n^2 \left(\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} - \mathrm{e}^{-\frac{3}{2n}}\right)$.

4. Calcolare, se esiste, $\lim_{n\to+\infty} a_n$:

(a) $a_n = n^3 \left(\sin \frac{3}{n} - 3\tan \frac{1}{n} + \mathrm{e}^{-5n}\right)$;

(b) $a_n = n^4 \left(\cos\left(\frac{1}{n}\right) - \frac{1}{1+\frac{1}{2n^2}}\right)$;

(c) $a_n = \dfrac{\log(1+\frac{1}{n}) - \frac{1}{n}}{\log(n^2+3) - \log(n^2+2)}$;

(d) $a_n = \left(\sqrt[n]{2} + \tan \frac{2}{n}\right)^n$.

5. Calcolare, se esiste, $\lim_{n\to+\infty} a_n$:

$a)$ $a_n = \mathrm{e}^{2n} \left(1 - \frac{1}{n^2}\right)^{n^3}$;

$b)$ $a_n = n^3 \left(\sinh \frac{1}{n} - \frac{1}{n}\mathrm{e}^{\frac{3}{n}} - \frac{3}{n^2}\right)$;

$c)$ $a_n = n^5 \left(\sin \frac{1}{n^2} - \frac{1}{n^2} - \frac{1}{6n^4}\right)$;

$d)$ $a_n = \left(\mathrm{e}^{\frac{1}{n}} - \sin \frac{1}{n}\right)^n$;

$e)$ $a_n = n^2 \left(\mathrm{e}^{\frac{2}{n}} \sinh \frac{1}{n} - \sin \frac{1}{n}\right)$;

$f)$ $a_n = n^\alpha \left(\sqrt[3]{n^3 - n} - n\right)$, $\alpha \in \mathbb{R}$.

6. Calcolare i seguenti limiti di successioni.

- $\lim_{n\to+\infty} n\left(2+n^2\sin\frac{1}{n}-\sqrt{n^2+4n+5}\right)$; $\qquad -\frac{2}{3}$
- $\lim_{n\to+\infty} n^3\left(3\tan\frac{1}{n}-\sin\frac{3}{n}-e^{-3n}\right)$ $\qquad \frac{11}{2}$
- $\lim_{n\to+\infty} n^2\left(\left(1+\frac{2}{n}\right)^n-e^2\left(1-\frac{2}{n}\right)\right)$ $\qquad \frac{14}{3}e^2$
- $\lim_{n\to+\infty} n^4\left(\cos\frac{1}{n}-\sqrt{1-\frac{1}{n^2}}\right)$ $\qquad \frac{1}{6}$

7. Determinare $a, b, c \in \mathbb{R}$ tali che

$\exp\left(\sqrt{n+2}-\sqrt{n}\right)-1+\cos\left(\frac{1}{\sqrt[4]{n}}\right)=a+\frac{b}{\sqrt{n}}+\frac{c}{n}=o(n^{-1})$.

$a=1,\ b=\frac{1}{2},\ c=\frac{13}{24}$

8. Al variare di $\alpha \in \mathbb{R}$ calcolare

$$\lim_{n\to+\infty} n^\alpha\left(e^{1/2n}\left(1+\sin\frac{1}{n}\right)^n-e\right).$$