Limiti notevoli

Utilizzeremo la seguente notazione:

$0 \neq \varepsilon_n \to 0$

$|z_n| \to +\infty$

$a, p, q \in \mathbf{R}$, $1 \neq a > 0$

$x_n \ll y_n \equiv \frac{x_n}{y_n} \to 0$

$x_n \sim y_n \equiv \frac{x_n}{y_n} \to 1$

1. Potenze e successioni geometriche:

(a) $n^p \to \begin{cases} 0 & \textit{se } p < 0 \\ 1 & \textit{se } p = 0 \\ +\infty & \textit{se } p > 0 \end{cases}$

(b) $q^n \to \begin{cases} 0 & \textit{se } |q| < 1 \\ 1 & \textit{se } q = 1 \\ +\infty & \textit{se } q > 1 \\ \nexists & \textit{se } q \leq -1 \end{cases}$

2. Gerarchia di infiniti: *Per* $a > 1$, $p > 0$ *si ha*

(a) $\log_a n \ll n^p \ll a^n \ll n! \ll n^n$, *oppure, più in generale:*
$(\log_a z_n)^q \ll (z_n)^p \ll a^{z_n} \ll [z_n]! \ll (z_n)^{z_n}$, *se* $z_n \to +\infty$.

(b) $(\varepsilon_n)^p \cdot |\log \varepsilon_n|^q \to 0$ *se* $\varepsilon_n \to 0_+$.

3. "Figli di e":

(a) $\left(1 + \frac{p}{z_n}\right)^{z_n} \to e^p$ *(in particolare,* $(1 + (1/n))^n \to e$*)*

(b) $(1 + p\varepsilon_n)^{1/\varepsilon_n} \to e^p$

(c) $\frac{\log_a(1+\varepsilon_n)}{\varepsilon_n} \to \frac{1}{\log a}$ *(in particolare,* $\frac{\log(1+\varepsilon_n)}{\varepsilon_n} \to 1$*; equivalentemente:* $\frac{\log t_n}{t_n - 1} \to 1$ *se* $1 \neq t_n \to 1$*)*

(d) $\frac{a^{\varepsilon_n}-1}{\varepsilon_n} \to \log a$ *(in particolare,* $\frac{e^{\varepsilon_n}-1}{\varepsilon_n} \to 1$*)*

(e) $\frac{(1+\varepsilon_n)^p - 1}{\varepsilon_n} \to p$

4. Altri limiti notevoli:

(a) $\frac{f(\varepsilon_n)}{\varepsilon_n} \to 1$ *per le seguenti funzioni* f*:* $\sin x$, $\tan x$, $\arcsin x$, $\arctan x$, $\mathrm{Sh}(x)$, $\mathrm{Th}(x)$

(b) $\frac{1 - \cos \varepsilon_n}{\varepsilon_n^2} \to \frac{1}{2}$, $\frac{\mathrm{Ch}(\varepsilon_n) - 1}{\varepsilon_n^2} \to \frac{1}{2}$.

5. Formula di Stirling: $n! \sim \frac{n^n \sqrt{2\pi n}}{e^n}$ *(in particolare:* $\log n! \sim n \log n$*)*