Category Theory

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1. Let $\mathbb{C}$ be a category with finite limits and $\mathbb{L}$ be a localization of $\mathbb{C}$, with $J : \mathbb{L} \to \mathbb{C}$ representing the inclusion functor and $L : \mathbb{C} \to \mathbb{L}$ its left adjoint.

   (i) Show that $\mathbb{L}$ has finite limits.

   (ii) Show that terminal objects in $\mathbb{C}$ and $\mathbb{L}$ are isomorphic.

   (iii) Show that, if $\mathbb{C}$ has a subobject classifier, the same happens for $\mathbb{L}$.

2. Let $\mathbb{C}$ be a pointed category with finite limits. Show that given a commutative diagram

$$\begin{array}{ccc} A & \xrightarrow{k} & B \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle f} \\ C & \xrightarrow[g]{} & D \end{array}$$

   if it is a pullback then the restriction of h to the kernels of k and g is an isomorphism.

   Is it the vice versa true?

3. The forgetful functor $U : \mathrm{Top} \to \mathrm{Set}$ from the category of topological spaces to the category of sets is a left adjoint. If $I : \mathrm{Set} \to \mathrm{Top}$ denotes its right adjoint, describe the category of the algebras of the associated monad $T = IU$.