Category Theory

July, 14 2016

1. Let $Sub(Top)$ be the category whose objects are pairs of topological spaces (X, A) with A subspace of X and morphisms are continuous maps $f : X \to X'$ with $f(A) \subseteq A'$. Show the forgetful functor $U : Sub(Top) \to Top$ given by $U(X, A) = X$ has a left adjoint F and a right adjoint H. Study the algebras of the corresponding monads UF and HU. Is U a monadic functor? Is H a monadic functor?

2. Let $\mathbf{A}$ be an abelian category, and H, K subobjects of G.

   (i) Show that there exist in $\mathbf{A}$ both the inf $H \wedge K$ and the sup $H \vee K$ in the poset Sub (G) e describe how to construct them.

   (ii) Denoting with $K/H \wedge K$, $H \vee K/H$ the corresponding quotients (cokernels), show they are isomorphic.

3. If I denotes a set, show that the category Set/I, whose objects are functions $\alpha : X \to I$ with codomain I and a morphism $f : \alpha \to \alpha'$ is a function $f : X \to X'$ with $\alpha' f = \alpha$ has a subobject classifier.