Category Theory

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1. Let $\mathbb{C}$ be an elementary topos and let $\mathbb{B}$ be a coreflective subcategory of $\mathbb{C}$, closed under the formation of finite limits in $\mathbb{C}$. Is $\mathbb{B}$ still an elementary topos?

2. Let

$$\begin{array}{ccc} A & \xrightarrow{k} & B \\ {\scriptstyle h}\big\downarrow & & \big\downarrow{\scriptstyle f} \\ C & \xrightarrow[g]{} & D \end{array}$$

   be a pushout diagram in any category $\mathbb{C}$.

   (i) Show that if k is a regular epimorphism, so is g.

   (ii) Show in case $\mathbb{C}$ is abelian, that if k is a monomorphism, so is g.

3. Given a group G, let Set^G be the category of G-sets. For any G-set X, let $\Pi(X)$ denote the set of orbits of X.

   (i) Show that Π gives rise to a functor $\pi : Set^G \to Set$.

   (ii) Is Π a left adjoint? If so, find a right adjoint $G : Set \to Set^G$ and then describe the algebras of the associated monad $G\Pi$.