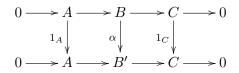
## **Category Theory**

## February 25, 2015

- 1. Let  $\mathbb{C}$  be a category with finite sums. Given a fixed object C in  $\mathbb{C}$ , for any other object X, let T(X) = X + C,  $\eta_X : X \to X + C$  be the canonical arrow into the sum and  $\mu_X : X + C + C \to X + C$  be  $1_X + \nabla_C$ , where  $\nabla_C = [1_C, 1_C]$  is the codiagonal of C.
  - (i) Show that the above assignments give rise to a monad  $(T, \eta, \mu)$  on  $\mathbb{C}$ .
  - (ii) Describe the category of the algebras for such a monad in the case  $\mathbb{C} = Set$  and C = 1, the terminal object.
- 2. Let



be a morphism of exact sequences in an abelian category  $\mathbb{C}$ Show that  $\alpha$  is an isomorphism.

- 3. Let  $\mathbb{C}$  be a small category with pullbacks.
  - (i) Show that the collection of all non-empty sieves on objects of C is a Grothendieck topology
  - (ii) Show that in case of  $\mathbb{C} = \bullet \rightrightarrows \bullet$  the above assertion is not true.
- 4. (Just if you want to do it!) Let  $\mathbb{C}$  be a category with a subobject classifier  $t: 1 \to \Omega$ . If A is a suboject of B, and B is a subobject of X, so that A is also a suboject of X, how are related the characteristic arrows  $\chi_A: B \to \Omega$  and  $\overline{\chi}_A: X \to \Omega$ ?