

# Analisi Matematica III

## Integrale multiplo di Riemann

1] Calcolare, se esistono, i seguenti integrali doppi.  $\int_D f$  dove

a)  $f(x, y) = x^2 + y$  e  $D = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, x + y \leq 1\}$  [1/4]

b)  $f(x, y) = \frac{x}{1+y}$  e  $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, x^2 \leq y \leq x\}$  [3/4 - log 2]

c)  $f(x, y) = y$  e  $D = \{(x, y) \in \mathbb{R}^2 : (x-1)^2 + y^2 \geq 1, (x-2)^2 + y^2 \leq 1, y \geq 0\}$  [11/24]

d)  $f(x, y) = x + y$  e  $D = \{(x, y) \in \mathbb{R}^2 : x \geq y^2 - 1, y \geq 2x - 1\}$  [125/192]

e)  $f(x, y) = (x-y)y$  e

$$D = \{(x, y) \in \mathbb{R}^2 : (x-y)^2 + y^2 \geq 1, 0 \leq x \leq 2, y \geq 0, x-y \geq 0\} \quad [13/24]$$

f)  $f(x, y) = \frac{y}{x^2 + y^2}$  e  $D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4, y \geq 0, \}$  [2]

g)  $f(x, y) = y^2$  e  $D = \{(x, y) \in \mathbb{R}^2 : 1 \geq x^2 + y^2, 0 \leq y \leq 2 - (x-1)^2, \}$  [ $\frac{1}{16}\pi$ ]

h)  $f(x, y) = x - y$  e  $D = \left\{(x, y) \in \mathbb{R}^2 : (x-3)^2 + (y-\sqrt{3})^2 \leq 1, x \geq 3, x \geq \sqrt{3}y\right\}$   
 $\left[\frac{1}{6}\sqrt{3} + \frac{1}{2} + \pi(3 - \sqrt{3})\right]$

i)  $f(x, y) = xy$  e  $D = \{(x, y) \in \mathbb{R}^2 : x^2 - x + y^2 \leq 0, y \geq x\}$  [ $\frac{1}{192}$ ]

l)  $f(x, y) = \frac{x^2}{x^2 + y^2}$  e  $D = \{(x, y) \in \mathbb{R}^2 : x \geq |y|, x \leq 1\}$  [ $\frac{1}{4}\pi$ ]

m)  $f(x, y) = \frac{1}{\sqrt{4-x^2-y^2}}$  e  $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2x\}$  [2π - 4]

n)  $f(x, y) = \exp(y^2)$  e  $D$  il triangolo di vertici  $(0, 0)$ ,  $(0, 3)$  e  $(2, 3)$   $\left[\frac{1}{3}(e^9 - 1)\right]$

o)  $f(x, y) = xe^{2y}$  e  $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \geq 1, 0 \leq x \leq 1, 0 \leq y \leq 1\}$  [ $(e^2 - 1)/8$ ]

**2]** Calcolare, se esistono, i seguenti integrali tripli.  $\int_D f$  dove

a)  $f(x, y, z) = y^2 x$  e  $D = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq 2, 0 \leq y \leq z, 0 \leq x \leq yz\}$  [16/5]

b)  $f(x, y, z) = z$  e  $D = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x, 0 \leq y, 0 \leq z, x + y + z \leq 1\}$  [1/24]

c)  $f(x, y, z) = \frac{1}{1+xy}$  e

$$D = \{(x, y, z) \in \mathbb{R}^3 : y^2 - 1 \leq x, x - 1 \leq y, xy - z + 1 \geq 0, z \geq 0\} \quad [9/2]$$

d)  $f(x, y, z) = z + 1$  e  $D = \{(x, y, z) \in \mathbb{R}^3 : z \leq 4(x^2 + y^2), z \leq 1, z \geq x^2 + y^2\}$  [5π/8]

e)  $f(x, y, z) = z$  e  $D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + (z + 1)^2 \leq 4, 3z \leq x^2 + y^2 - 3\}$  [-43π/4]

f)  $f(x, y, z) = y$  e  $D = \{(x, y, z) \in \mathbb{R}^3 : (x - 1)^2 + y^2 \leq 1, z \leq x^2 + y^2, z \geq 0\}$  [0]

g)  $f(x, y, z) = \frac{xe^z}{y}$  e

$$D = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq 1, x^2 \leq y, 0 < z \leq y \leq \min(1, 2x^2)\} \quad [(e - 2)/4]$$

**3]** Calcolare il volume dei seguenti solidi

a)  $\Omega = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq z \leq x - 2y\}$

b)  $\Omega = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1, x^2 \geq y^2, -1 \leq z \leq 1\}$

c)  $\Omega = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 4, -(x^2 + y^2) \leq z, z\sqrt{x^2 + y^2} \leq 1, z \leq 1\}$