I Introduction to financial markets and options


II Brief introduction to stochastic processes


III Discrete time models and the First Fundamental Theorem of Asset Pricing

Multi-period markets. Hedging strategies and replicable contingent claims. Discounted prices and the value process. Definition of Arbitrage opportunity and equivalent characterizations. Equivalent martingale measures and no arbitrage pricing. The first fundamental theorem of asset pricing: NA is equivalent to the existence of an equivalent martingale measure. The second fundamental theorem of asset pricing: complete markets. The pricing formula of replicable claims. The set \( K \) and the cone \( C \) of super replicable claims. The Kreps-Yan Theorem in \( L^1 \). The proof of the 1FTAP for a finite probability space. The proof of the 1FTAP for a general probability space in one time period \([t-1,t]\). Conditional change of measure. Extension of the proof to the multi-period case. The No Arbitrage and the No Free Lunch with Vanishing Risk conditions and the strong/weak closure of \( C \).

IV Complete and Incomplete Markets

Brief introduction to convex analysis: polar and bipolar of a convex cone. Characterization of the set \( M \) of martingale measures, as the (normalized) polar set of \( C \). Density of the set of equivalent martingale measures in the set of absolutely continuous martingale measures. Characterization of the cone \( C \) of integrable super replicable claims as the polar of \( M^a \).

Incomplete markets: the pricing problem. The super replication cost and the no arbitrage interval. Duality theorem for the super replication cost under the assumption that \( C \) is closed in \( L^1 \).

(M.Burzoni) The second fundamental theorem of asset pricing and its proof. Pricing and hedging in the binomial model. Examples and pricing of particular contingent claims.
V Brief introduction to stochastic processes

On the Brownian Motion (BM) and continuous time martingales. The exponential martingale. The BM paths are not of finite variation. Quadratic variation of the BM. The stochastic integral w.r.t. the BM and the martingale property. Ito processes and Ito formula. Novikov Lemma and Girsanov Theorem. The predictable representation property. Examples of stochastic differential equations.

VI Continuous time markets

The BS model and the no arbitrage principle. Existence of an equivalent martingale measure. The BS formula, its probabilistic and analytic proofs. Study of the BS partial differential equation.


Pricing of Perpetual American Put options.
Contingent claim pricing in incomplete continuous time markets: pricing in models with one tradable risky asset and one non-tradable risky asset; the partial differential equation and the market price of risk.

Suggested bibliography

For an introduction to the course you can look at Part I, Section 1 and 2 of the following book.


Elements of probability and discrete stochastic processes.

2) D. Williams, Probability with martingales. Cambridge University Press.

Discrete time financial markets.

4) H. Foellmer and A. Schied, Stochastic Finance (an introduction in discrete time), de Gruyter.

Continuous time financial markets.

5) S. Shreve, Stochastic Calculus for Finance II. Springer.
6) T. Bjork, Arbitrage Theory in Continuous Time, Oxford University Press.