On the second maximal groups with respect to the sum of element orders

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Let G be a finite group and consider the function ψ defined by

$$\psi(G) = \sum_{x \in G} o(x),$$

where $o(x)$ denotes the order of the element $x \in G$.

In [1] H. Amiri, S.M. Jafarian Amiri and M. Isaacs proved that if G has order n and C_n denotes the cyclic group of order n, then $\psi(G) \leq \psi(C_n)$, and $\psi(G) = \psi(C_n)$ if and only if $G \simeq C_n$.
Other results have been obtained by H. Amiri, S.M. Jafarian Amiri, M. Amiri, Y. Marefat, A. Iranmanesh, A. Tehranian, R. Shen, G. Chen and C. Wu.

We will report some new results concerning the function ψ, jointly obtained with Marcel Herzog and Mercede Maj in the papers [2], [3], [4], [5]. In particular we will present some better upper bounds for $\psi(G)$ when G is not cyclic, and some results on the structure of G assuming some bounds on $\psi(G)$.

References

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