ANALISI DI FOURIER– A.A. 2016/17
Corsi di Laurea in Matematica
Esercizi

I.1. Show that $C_0(\mathbb{R}^n)$ is a Banach space with the sup norm. Show that $C_c(\mathbb{R}^n)$ is dense in $C_0(\mathbb{R}^n)$, and that functions in $C_0(\mathbb{R}^n)$ are uniformly continuous.

I.2. Prove Generalized Hölder's inequality, (Exercise 4 in the notes).

I.3. Show that if $\varphi \in \mathcal{S}(\mathbb{R}^n)$ and $\int \varphi = 1$, then for $\psi \in \mathcal{S}$, $\varphi_t * \psi \to \psi$ in $\mathcal{S}$.

I.4. Prove inequality (1.7) in the notes.

I.5. (i) Verify that $u(x) = \log|x|$ defines a tempered distribution and that (the distributional derivative) $u'(x)$ equals p.v.$\frac{1}{x}$. (ii) Let $v_m(x) = \frac{1}{1+i/m}$, $x \in \mathbb{R}$, $m = 1, 2, \ldots$. Find the limit in of the sequence v_m in $\mathcal{S}'$.