

Analisi Matematica 1
Formule di McLaurin di Funzioni Elementari

Supponiamo che $x \rightarrow 0$. Allora

- $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^k}{k!} + o(x^k)$
- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2k+2})$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2k+1})$
- $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2k+1}}{(2k+1)!} + o(x^{2k+2})$
- $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2k}}{(2k)!} + o(x^{2k+1})$
- $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{k-1} \frac{x^k}{k} + o(x^k)$
- $(1+x)^\alpha = 1 + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \cdots + \binom{\alpha}{k}x^k + o(x^k)$

dove, per $\alpha \in \mathbb{R}$ e $k \in \mathbb{N}$, $\binom{\alpha}{k}$ è il coefficiente binomiale generalizzato così definito

$$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!}.$$

In particolare, per $\alpha = -1$ e $\alpha = \frac{1}{2}$ si ottiene

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^k x^k + o(x^k)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + o(x^4)$$

- $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2k+2})$
- $\arcsin x = x + \frac{x^3}{6} + \frac{3}{40}x^5 + o(x^6)$
- $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + o(x^6)$