Analisi Matematica 1

Sviluppi notevoli (Successioni)

Supponiamo che $\varepsilon_n \neq 0$ e $\varepsilon_n \to 0$. Allora

- $e^{\varepsilon_n} = 1 + \varepsilon_n + \frac{(\varepsilon_n)^2}{2!} + \cdots + \frac{(\varepsilon_n)^k}{k!} + o\Big((\varepsilon_n)^k\Big)$

- $\sin \varepsilon_n = \varepsilon_n - \frac{(\varepsilon_n)^3}{3!} + \frac{(\varepsilon_n)^5}{5!} + \cdots + (-1)^k \frac{(\varepsilon_n)^{2k+1}}{(2k+1)!} + o\Big((\varepsilon_n)^{2k+2}\Big)$

- $\cos \varepsilon_n = 1 - \frac{(\varepsilon_n)^2}{2!} + \frac{(\varepsilon_n)^4}{4!} + \cdots + (-1)^k \frac{(\varepsilon_n)^{2k}}{(2k)!} + o\Big((\varepsilon_n)^{2k+1}\Big)$

- $\sinh \varepsilon_n = \varepsilon_n + \frac{(\varepsilon_n)^3}{3!} + \frac{(\varepsilon_n)^5}{5!} + \cdots + \frac{(\varepsilon_n)^{2k+1}}{(2k+1)!} + o\Big((\varepsilon_n)^{2k+2}\Big)$

- $\cosh \varepsilon_n = 1 + \frac{(\varepsilon_n)^2}{2!} + \frac{(\varepsilon_n)^4}{4!} + \cdots + \frac{(\varepsilon_n)^{2k}}{(2k)!} + o\Big((\varepsilon_n)^{2k+1}\Big)$

- $\log(1 + \varepsilon_n) = \varepsilon_n - \frac{(\varepsilon_n)^2}{2} + \frac{(\varepsilon_n)^3}{3} + \cdots + (-1)^{k-1} \frac{(\varepsilon_n)^k}{k} + o\Big((\varepsilon_n)^k\Big)$

- $(1 + \varepsilon_n)^\alpha = 1 + \binom{\alpha}{1}\varepsilon_n + \binom{\alpha}{2}(\varepsilon_n)^2 + \cdots + \binom{\alpha}{k}(\varepsilon_n)^k + o\Big((\varepsilon_n)^k\Big)$

  dove, per $\alpha \in \mathbb{R}$ e $k \in \mathbb{N}$, $\binom{\alpha}{k}$ è il coefficiente binomiale generalizzato così definito

  $$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!}.$$

  In particolare, per $\alpha = -1$ e $\alpha = \frac{1}{2}$ si ottiene

  $$\frac{1}{1+\varepsilon_n} = 1 - \varepsilon_n + (\varepsilon_n)^2 - (\varepsilon_n)^3 + \cdots + (-1)^k (\varepsilon_n)^k + o\Big((\varepsilon_n)^k\Big)$$

  $$\sqrt{1+\varepsilon_n} = 1 + \tfrac{1}{2}\varepsilon_n - \tfrac{1}{8}(\varepsilon_n)^2 + \tfrac{1}{16}(\varepsilon_n)^3 - \tfrac{5}{128}(\varepsilon_n)^4 + o\Big((\varepsilon_n)^4\Big)$$

- $\arctan \varepsilon_n = \varepsilon_n - \frac{(\varepsilon_n)^3}{3} + \frac{(\varepsilon_n)^5}{5} + \cdots + (-1)^k \frac{(\varepsilon_n)^{2k+1}}{2k+1} + o\Big((\varepsilon_n)^{2k+2}\Big)$

- $\arcsin \varepsilon_n = \varepsilon_n + \frac{(\varepsilon_n)^3}{6} + \frac{3}{40}(\varepsilon_n)^5 + o\Big((\varepsilon_n)^6\Big)$

- $\tan \varepsilon_n = \varepsilon_n + \frac{(\varepsilon_n)^3}{3} + \frac{2}{15}(\varepsilon_n)^5 + o\Big((\varepsilon_n)^6\Big)$