Effective stability around the Cassini state in the spin-orbit problem — Supplementary Material —

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1 Explicit expansions

This is the supplementary material of our paper "Effective stability around the Cassini state in the spin-orbit problem". We report here the explicit expansions of two relevant Hamiltonian functions concerning the application of our methods to the largest moon of Saturn, Titan. We recall that the Titan physical parameters adopted here are reported in Table 1 of the paper.

In Table 2 we provide the approximation up to order 6 in $\left(\sqrt{U_1}\,,\ \sqrt{U_3}\right)$ of the Hamiltonian

$$H^{(0)}(U,\,u) = \omega_u \cdot U + \sum_{j>0} H_j^{(0)}(U,\,u)\ .$$

For all the details see Eq. (12) in Subsection 3.1 of the paper.

In Table 3 we report the truncated normal form, up to order 12, of the Hamiltonian

$$H^{(10)}(U,\,u) = Z_0(U) + \ldots + Z_{10}(U)\ .$$

For all the details see Eq. (13) in Subsection 3.2 of the paper.

k_1	k_3	h_1	h_3	C_{k_1,k_3,h_1,h_3}
2	0	0	0	2.6909586461447121e+00
0	2	0	0	2.3752838852525892e−02
3	0	1	0	2.8594188845630538e−07
3	0	3	0	−2.8663839806645031e−07
2	1	0	1	−3.9900442186763694e−03
2	1	2	1	2.0115820266225605e−03
2	1	2	−1	1.9844544796434136e−03
1	2	1	0	−3.9654615580802079e−03
1	2	1	2	−9.6201937136893220e−02
1	2	1	−2	9.2864206735268406e−02
0	3	0	1	−2.1875957544878832e+00
0	3	0	3	−2.1875281225825116e+00
4	0	0	0	−3.5981000075184902e+01
4	0	2	0	4.7974666723421883e+01
4	0	4	0	−1.1993666648088578e+01
3	1	1	1	2.6053340451744956e−04
3	1	1	−1	2.6484192513816327e−04
3	1	3	1	−2.6425921321490659e−04
3	1	3	−1	−2.6243716227760538e−04
2	2	0	0	−9.7728494019965051e−01
2	2	2	0	9.7818976319532136e−01
2	2	0	2	3.9495665511463156e−04
2	2	2	2	8.0893976940895670e−03
2	2	2	−2	−7.6926236488083525e−03
1	3	1	1	9.3498639124226450e+00
1	3	1	−1	−8.0624608422846151e+00
1	3	1	3	3.2384127644395271e+00
1	3	1	−3	−2.5658174415814305e+00
0	4	0	0	3.0218984800687474e+02
0	4	0	2	5.3724313061153964e+02
0	4	0	4	1.6788922342336792e+02
5	0	1	0	−5.1011112841311461e−06
5	0	3	0	7.6587974130777196e−06
5	0	5	0	−2.5577171444941836e−06
4	1	0	1	1.0674490540072289e−01
4	1	2	1	−7.1687650600875175e−02
4	1	2	−1	−7.0720844676127062e−02
4	1	4	1	1.8073858124807902e−02
4	1	4	−1	1.7590012539509887e−02
3	2	1	0	8.1475227651977580e−02
3	2	3	0	−8.1662007049956464e−02
3	2	1	2	5.0895664301842922e+00
3	2	1	−2	−5.0209955300671867e+00
3	2	3	2	−1.7203515148108348e+00
3	2	3	−2	1.6516143196460535e+00
2	3	0	1	8.9841923179718094e+01
2	3	2	1	−4.5852751651956467e+01
2	3	2	−1	−4.4399243190345018e+01
2	3	0	3	8.9956144756923081e+01
2	3	2	3	−4.5717046579116989e+01
2	3	2	−3	−4.4416942499235240e+01
1	4	1	0	−2.0615251574082052e+02
1	4	1	2	−6.8313914281171947e+02
1	4	1	−2	3.8584428706381976e+02
1	4	1	4	−5.7921834008815381e+02
1	4	1	−4	4.8985128801338442e+02
0	5	0	1	−1.5461772756469477e+05
0	5	0	3	−8.3492764779376870e+04
0	5	0	5	−2.1646696433523161e+04
6	0	0	0	4.2764846157177988e+02
6	0	2	0	−6.4147269177557803e+02
6	0	4	0	2.5658907601050851e+02
6	0	6	0	−4.2764845806704066e+01
5	1	1	1	−4.5429638238788745e−03
5	1	1	−1	−4.8289177490204588e−03
5	1	3	1	7.0077095932897480e−03
5	1	3	−1	7.0636366975078734e−03
5	1	5	1	−2.3616109883692706e−03
5	1	5	−1	−2.3379120133985112e−03
4	2	0	0	2.6135281836972304e+01
4	2	2	0	−3.4859420990550618e+01
4	2	0	2	−8.1109338627207948e−03
4	2	4	0	8.7241757459888358e+00
4	2	2	2	−2.8125246787788366e−01
4	2	2	−2	2.8123653021431078e−01
4	2	4	2	1.4481621676353099e−01
4	2	4	−2	−1.3665648133732658e−01
3	3	1	1	−4.7876070212199522e+02
3	3	1	−1	4.5234340210256363e+02
3	3	3	1	1.6855594903979667e+02
3	3	3	−1	−1.4203319254260725e+02
3	3	1	3	−1.6204459162566110e+02
3	3	1	−3	1.4823323305576781e+02
3	3	3	3	5.8741087168690669e+01
3	3	3	−3	−4.4887536020189103e+01
2	4	0	0	−1.2400056754065656e+04
2	4	2	0	1.2478337341813696e+04
2	4	0	2	−2.2059673723489443e+04
2	4	2	2	1.1241863783933961e+04
2	4	2	−2	1.0927300951123980e+04
2	4	0	4	−6.8952167246115914e+03
2	4	2	4	3.5640063900143227e+03
2	4	2	−4	3.3621009146477213e+03
1	5	1	1	1.3106747223481991e+05
1	5	1	−1	−1.6609533001415599e+04
1	5	1	3	1.6580986386734922e+05
1	5	1	−3	−1.0494128560577944e+05
1	5	1	5	5.8709498567676383e+04
1	5	1	−5	−4.4666799904440653e+04
0	6	0	0	2.1454485357404534e+07
0	6	0	2	3.3036046316188511e+07
0	6	0	4	1.4467528535057161e+07
0	6	0	6	2.8099655679855803e+06

Table 2: Expansion of the Hamiltonian (12), up to order 6 in $\left(\sqrt{U_1}\,,\ \sqrt{U_3}\right)$. The generic term is represented in the form $C_{k_1,k_3,h_1,h_3}\,\sqrt{U_1}^{k_1}\sqrt{U_3}^{k_3}\,\cos(h_1 u_1 + h_3 u_3)$.

k_1	k_3	C_{k_1,k_3}	k_1	k_3	C_{k_1,k_3}
2	0	$2.6909586461447121e{+}00$	10	0	$-7.2189337551853480e{+}05$
0	2	$2.3752838852525892e{-}02$	8	2	$-1.8961235092414900e{+}04$
4	0	$-3.5981167642268979e{+}01$	6	4	$-8.4732126056309789e{+}02$
2	2	$-1.3448799121582549e{+}00$	4	6	$2.7263822555541992e{+}01$
0	4	$-1.3201245664845374e{-}02$	2	8	$-2.0806200186210938e{+}09$
6	0	$-4.8111401327259489e{+}02$	0	10	$5.0955389497500000e{+}11$
4	2	$-2.9854021077753146e{-}02$	12	0	$-3.7648814144418113e{+}07$
2	4	$-7.6379122810976696e{-}04$	10	2	$-1.3501710209223158e{+}06$
0	6	$5.8282166719436646e{-}06$	8	4	$-8.2404461873292923e{+}04$
8	0	$-1.6082804878843941e{+}04$	6	6	$3.1835101994540405e{+}08$
6	2	$-2.4062223903981055e{+}02$	4	8	$8.4296230702750000e{+}10$
4	4	$1.6882042837096378e{-}02$	2	10	$-9.9730555587680000e{+}13$
2	6	$4.6446472406387329e{-}03$	0	12	$1.6290727212810240e{+}16$
0	8	$1.2622881955383301e{+}07$			

Table 3: Expansion of the Hamiltonian in normal form, up to order 12 in $\left(\sqrt{U_1}\,,\ \sqrt{U_3}\right)$. The generic term is represented in the form $C_{k_1,k_3}\,\sqrt{U_1}^{k_1}\sqrt{U_3}^{k_3}$.