

Argomento 4

Suggerimenti

Suggerimenti Ex. 4.2

$$1. \frac{5-2x}{3x-7} = \frac{-2x \left(\frac{5}{-2x} + 1 \right)}{3x \left(1 - \frac{7}{3x} \right)} = \dots \text{oppure, per } x \rightarrow +\infty, 5-2x \sim -2x \text{ e } 3x-7 \sim 3x.$$

$$15. \sqrt{x^3-1} - \sqrt{x^2-x} = \sqrt{x^3} \left(\sqrt{1 - \frac{1}{x^3}} - \sqrt{\frac{1}{x} - \frac{1}{x^2}} \right) = \dots \text{oppure, per } x \rightarrow +\infty,$$

$$\sqrt{x^3-1} = x^{\frac{3}{2}} + o(x^{\frac{3}{2}}), \sqrt{x^3-1} = x^{\frac{3}{2}} + o(x^{\frac{3}{2}}), \sqrt{x^2-x} = x + o(x), \text{ perciò}$$

$$\sqrt{x^3-1} - \sqrt{x^2-x} = x^{\frac{3}{2}} + o(x^{\frac{3}{2}}) - x + o(x) = x^{\frac{3}{2}} + o(x^{\frac{3}{2}}) \sim x^{\frac{3}{2}},$$

perchè anche x e ogni $o(x)$ sono trascurabili rispetto a $x^{\frac{3}{2}}$.

$$16. \sqrt{x^2+2} - x = \frac{\sqrt{x^2+2} - x}{\sqrt{x^2+2} + x} (\sqrt{x^2+2} + x) = \dots$$

Suggerimenti Ex. 4.4

$$1. \frac{2x^4 + x^2 + 5x}{x^5 + 4x^3 - x} = \frac{5x \left(\frac{2}{5}x^3 + \frac{1}{5}x^2 + 1 \right)}{-x(-x^4 - 4x^2 + 1)} = \dots \text{oppure per } x \rightarrow 0$$

$$2x^4 + x^2 + 5x = 5x + o(x) \sim 5x \text{ e } x^5 + 4x^3 - x = -x + o(x) \sim -x.$$

Suggerimenti Ex. 4.5

$$1. \lim_{x \rightarrow +\infty} \frac{\frac{1}{x^2}}{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{x}{x^2} = \dots$$

$$4. \lim_{x \rightarrow +\infty} \frac{\frac{x+3}{x^2-2}}{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x^2}}{\frac{1}{x}} = \dots$$

Suggerimenti Ex. 4.6

$$7. \lim_{x \rightarrow 0} \frac{e^{3x} - 1 - 4x}{x} = \lim_{x \rightarrow 0} \frac{3x + o(x) - 4x}{x} = \lim_{x \rightarrow 0} \frac{-x + o(x)}{x} = \dots$$

$$12. \lim_{x \rightarrow 0^+} \frac{\sqrt[3]{x+1} - 1}{\sqrt{x} - \log(1+x)} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{3}x}{\sqrt{x} - x + o(x)} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{3}x}{\sqrt{x}}, \text{ perchè, per } x \rightarrow 0^+,$$

x e $o(x)$ sono trascurabili rispetto a \sqrt{x} .

Suggerimenti Ex. 4.7

10. Per $x \rightarrow 0^+$, $\sqrt{x} + (\sqrt{x+1} - 1) = x^{\frac{1}{2}} + \frac{1}{2}x + o(x) \sim x^{\frac{1}{2}}$ perchè x e ogni $o(x)$ sono trascurabili rispetto a $x^{\frac{1}{2}}$ (infinitesimi di ordine superiore).

12. $x^2(\sqrt{x^4+1} - 1) \sim x^2 \frac{1}{2}x^4 \sim \frac{1}{2}x^6$.

Suggerimenti Ex. 4.8

1. $\lim_{x \rightarrow 0} \frac{x^3 - \log(1+x)}{(e^x - 1)x} = \lim_{x \rightarrow 0} \frac{x^3 - (x + o(x))}{x^2 + o(x^2)} = \lim_{x \rightarrow 0} \frac{-x}{x^2} = \dots$

Suggerimenti Ex. 4.9

1. $\lim_{x \rightarrow +\infty} x \sin\left(\frac{1}{x}\right) = \lim_{x \rightarrow +\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \dots$

3. $\lim_{x \rightarrow +\infty} x^2 \log\left(\frac{x^2 - 4}{x^2}\right) = \lim_{x \rightarrow +\infty} x^2 \log\left(1 - \frac{4}{x^2}\right) = (-4) \lim_{x \rightarrow +\infty} \frac{\log\left(1 - \frac{4}{x^2}\right)}{\frac{-4}{x^2}} = \dots$

Suggerimenti Ex. 4.11

1. $\lim_{x \rightarrow +\infty} x^{\frac{1}{x}} = \lim_{x \rightarrow +\infty} e^{\frac{1}{x} \log x} = \lim_{x \rightarrow +\infty} \frac{\log x}{x} = 1$

7. $\lim_{x \rightarrow +\infty} \left(\frac{x+1}{x}\right)^{\frac{x^2+1}{2x+1}} = \lim_{x \rightarrow +\infty} e^{\frac{x^2+1}{2x+1} \log\left(1+\frac{1}{x}\right)} = \lim_{x \rightarrow +\infty} \frac{x^2+1}{2x+1} \log\left(1+\frac{1}{x}\right) = \lim_{x \rightarrow +\infty} \frac{x^2+1}{2x+1} \cdot \frac{1}{x} = \frac{1}{2}$.

8. $\lim_{x \rightarrow +\infty} \left(1 - \sin \frac{1}{x^2}\right)^{\frac{1}{\sin\left(\frac{1}{x^2}\right)}} = \lim_{x \rightarrow +\infty} e^{\frac{\log\left(1+\frac{1}{x^2}\right)}{\sin\left(\frac{1}{x^2}\right)}} = \lim_{x \rightarrow +\infty} \frac{\log\left(1 - \sin \frac{1}{x^2}\right)}{\sin\left(\frac{1}{x^2}\right)} = \lim_{x \rightarrow +\infty} -\frac{\sin \frac{1}{x^2}}{\sin \frac{1}{x^2}} = \dots$

Suggerimenti Ex. 4.12

$\lim_{x \rightarrow +\infty} x^\alpha (\sqrt[3]{x^4+x} - x) = \lim_{x \rightarrow +\infty} x^\alpha \sqrt[3]{x^4} = \dots$

Suggerimenti Ex. 4.13

22. $\lim_{x \rightarrow +\infty} (\sqrt{x^2+1} - \sqrt{x^2}) = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+1} - \sqrt{x^2}}{\sqrt{x^2+1} + \sqrt{x^2}} (\sqrt{x^2+1} + \sqrt{x^2}) =$
 $= \lim_{x \rightarrow +\infty} \frac{(x^2+1 - x^2) \sqrt[3]{x^2}}{\sqrt{x^2+1} + \sqrt{x^2}} = \dots$

Argomento 4

Soluzioni

Soluzioni Ex. 4.1

- | | | |
|-------------------------|-------------------------|-------------------------|
| 1. si, ordine superiore | 2. si, ordine superiore | 3. si, ordine superiore |
| 4. si, ordine inferiore | 5. si, ordine superiore | 6. si, stesso ordine |

Soluzioni Ex. 4.2

- | | | |
|--------------------|-------------------|------------------|
| 1. $-\frac{2}{3}$ | 2. $+\infty$ | 3. $\frac{4}{9}$ |
| 4. 0 | 5. 0 | 6. $\frac{1}{3}$ |
| 7. $\frac{2}{3}$ | 8. 2 | 9. 1 |
| 10. $-\frac{1}{3}$ | 11. -1 | 12. 2 |
| 13. 1 | 14. $\frac{1}{2}$ | 15. $+\infty$ |
| 16. 0 | 17. $\frac{1}{2}$ | 18. 0 |

Soluzioni Ex. 4.3

- | | | |
|-------------------------|----------------------|----------------------|
| 1. si, ordine superiore | 2. si, stesso ordine | 3. no |
| 4. si, ordine inferiore | 5. no | 6. si, stesso ordine |

Soluzioni Ex. 4.4

- | | | |
|--------------|--------------|------------------|
| 1. -5 | 2. 0 | 3. $+\infty$ |
| 4. $-\infty$ | 5. $-\infty$ | 6. $\frac{1}{4}$ |

Soluzioni Ex. 4.5

- | | | |
|-------------------------|----------------------|-------------------------|
| 1. si, ordine superiore | 2. si, stesso ordine | 3. si, ordine inferiore |
| 4. si, stesso ordine | 5. no | 6. si, stesso ordine |

Soluzioni Ex. 4.6

- | | | |
|------------------|--------------------|-------|
| 1. $\frac{2}{3}$ | 2. 0 | 3. 1 |
| 4. 1 | 5. $+\infty$ | 6. 0 |
| 7. 0 | 8. -1 | 9. 1 |
| 10. -1 | 11. $-\frac{1}{2}$ | 12. 0 |
| 13. 0 | 14. $\frac{1}{3}$ | 15. 0 |

Soluzioni Ex. 4.7

- | | | | |
|----------------------|---------------|----------------------|---------------------|
| 1. x^2 | 2. \sqrt{x} | 3. $-\frac{1}{2}x^6$ | 4. $\frac{1}{2}x^3$ |
| 5. $x^{\frac{9}{2}}$ | 6. $3x$ | 7. x^4 | 8. x |

Soluzioni Ex. 4.8

- | | | |
|---------------|-------------------|---------------|
| 1. non esiste | 2. 2 | 3. -15 |
| 4. 0 | 5. $-\frac{1}{2}$ | 6. non esiste |

Soluzioni Ex.4.9

- | | | |
|--------------|------|-------|
| 1. 1 | 2. 0 | 3. -4 |
| 4. $+\infty$ | 5. 0 | 6. 0 |

Soluzioni Ex. 4.10

- | | | |
|--------------|------|------|
| 1. $+\infty$ | 2. 1 | 3. 1 |
|--------------|------|------|

Soluzioni Ex. 4.11

- | | | |
|------|----------|----------------------|
| 1. 1 | 2. 1 | 3. $e^{\frac{1}{2}}$ |
| 4. 0 | 5. e^2 | 6. e^2 |

Soluzioni Ex. 4.12

$$\lim_{x \rightarrow +\infty} x^\alpha \left(\sqrt[3]{x^4 + x} - x \right) = \begin{cases} +\infty & \text{se } \alpha > -\frac{4}{3} \\ 1 & \text{se } \alpha = -\frac{4}{3} \\ +\infty & \text{se } \alpha < -\frac{4}{3} \end{cases}$$

Soluzioni Ex. 4.13

- | | | |
|---------------|-------------------|------------------|
| 1. $+\infty$ | 2. 0 | 3. $\sqrt{2}$ |
| 4. $+\infty$ | 5. $-\infty$ | 6. $\frac{8}{9}$ |
| 7. 0 | 8. $+\infty$ | 9. 0 |
| 10. $-\infty$ | 11. $\frac{1}{2}$ | 12. 5 |
| 13. $+\infty$ | 14. 0 | 15. 0 |
| 16. $-\infty$ | 17. 0 | 18. $-\infty$ |
| 19. $-\infty$ | 20. 0 | 21. non esiste |
| 22. 0 | 23. 2 | 24. $+\infty$ |