Equazioni differenziali, #1

Marco Vignati - Metodi matematici applicati alla chimica - L.M. in Chimica - 2015/16

Determinare la soluzione locale dei seguenti problemi di Cauchy.

I ordine, lineari

1] $\begin{cases} y' + \dfrac{y}{2\sqrt{t}} = 1 \\ y(1) = 0 \end{cases}$ **2]** $\begin{cases} y' = \dfrac{2t - y}{t} \\ y(2) = 1 \end{cases}$ **3]** $\begin{cases} y' = 1 + y\left(2t - \dfrac{1}{t}\right) \\ y(1) = 1/4 \end{cases}$

Sol.: $y\,(t) = 2\left(\sqrt{t} - 1\right)$; ; $y\,(t) = t - \dfrac{2}{t}$; $y\,(t) = \dfrac{1}{4t}\left[3e^{t^2-1} - 2\right]$

I ordine, variabili separabili

4] $\begin{cases} 2yy' = (y^2 + 4)t\cos t \\ y(0) = -1 \end{cases}$ **5]** $\begin{cases} 4y'y^3 = 2t + 1 \\ y(0) = 1 \end{cases}$ **6]** $\begin{cases} y' = 2t\left(y + \dfrac{1}{y}\right) \\ y(0) = -2 \end{cases}$

7] $\begin{cases} 3t^2yy' = 2 + y^2 \\ y(1) = -1 \end{cases}$ **8]** $\begin{cases} yy' = e^{2t+y^2} \\ y(0) = 2 \end{cases}$

Sol.: $y\,(t) = -\sqrt{5e^{(t\sin t + \cos t - 1)} - 4}$; $y\,(t) = \sqrt[4]{t^2 + t + 1}$; $y\,(t) = -\sqrt{5e^{2t^2} - 1}$;
$y\,(t) = -\sqrt{-2 + 3e^{\frac{2}{3}\left(1 - \frac{1}{t}\right)}}$; $y\,(t) = \sqrt{-\ln\left(1 + e^{-4} - e^{2x}\right)}$.

I ordine, Bernoulli

9] $\begin{cases} y' = -\dfrac{3y}{2t} + \dfrac{\log t}{y} \\ y(1) = -1 \end{cases}$, **10]** $\begin{cases} 2y' - \dfrac{y}{t+1} + ty^3 = 0 \\ y(0) = \sqrt{6} \end{cases}$

Sol.: $y\,(t) = -\dfrac{\sqrt{2}}{4t^2}\sqrt{t\left[9 - t^4 + 4t^4 \ln t\right]}$; $y\,(t) = \sqrt{\dfrac{6\,(1+t)}{2t^3 + 3t^2 + 1}}$.