Complementi di Matematica - Crema - 12/9/2012

Prova scritta

1) $p_2(x) = ax^2 + bx + c \quad ; \quad p'_2(x) = 2ax + b$

$p_2(0) = c = y_0$

$p'_2(x_1) = 2ax_1 + b = y_1$

$p_2(x_2) = ax_2^2 + bx_2 + c = y_2$

$$\underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 2x_1 & 1 & 0 \\ x_2^2 & x_2 & 1 \end{bmatrix}}_{A} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

$\det A = 2x_1x_2 - x_2^2 \neq 0 \quad x_2(2x_1 - x_2) \neq 0$

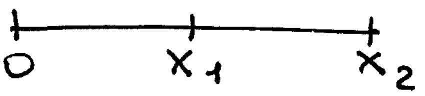
$x_2 \neq 0 \quad 2x_1 - x_2 \neq 0 \quad x_1 \neq \frac{x_2}{2}$

$\forall\, y_0 \ldots y_1 \ldots y_2$

Non può essere

$p_2(0) = y_0$

$p_2(0) = y_2$



NON DEVE ESSERE x_1 PUNTO MEDIO DI $0, x_2$

2] 2.1) $\det A = a \begin{vmatrix} a & \frac{2}{a} \\ \frac{a}{2} & a \end{vmatrix} = a(a^2 - 1) \neq 0 \quad \begin{matrix} a \neq 0 \\ a \neq \pm 1 \end{matrix}$

(sviluppando rispetto seconda riga/seconda colonna)

2.2) $\begin{cases} |a| > \left|\frac{2}{a}\right| \\ |a| > 0 \\ |a| > \left|\frac{a}{2}\right| \end{cases} \quad \begin{cases} |a|^2 > 2 \\ a \neq 0 \\ a \neq 0 \end{cases} \quad \boxed{a > \sqrt{2} \cup a < -\sqrt{2}}$

2.3) Simmetria : $\frac{2}{a} = \frac{a}{2} \quad a^2 = 4 \quad a = \pm 2$

Complementi di Matematica - Crema - 12/9/2012
Prova scritta

Caso $a=2$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$\det A_{11} = 2 > 0$

$\det A_{22} = \left|\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right| = 4 > 0$

$\det A_{33} = \det A = 2(4-1) = 6$

Caso $a=-2$

$$\begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

$a_{ii} < 0$
Non è D.P.

2.4) $\det \begin{bmatrix} a\lambda & 0 & \frac{2}{a} \\ 0 & a\lambda & 0 \\ \frac{a}{2} & 0 & a\lambda \end{bmatrix} = a\lambda\left(a^2\lambda^2 - 1\right) = 0$ $\quad \lambda = 0$, $\lambda = \pm\frac{1}{a}$

$\det(\lambda D + L + U)$

$\rho(B_J) = \frac{1}{|a|} < 1 \qquad |a| > 1$

2.5) $\det \begin{bmatrix} a\lambda & 0 & \frac{2}{a} \\ 0 & a\lambda & 0 \\ \frac{a}{2}\lambda & 0 & a\lambda \end{bmatrix} = a\lambda\left(a^2\lambda^2 - \lambda\right) = 0$ $\quad \lambda = 0$, $\lambda = \frac{1}{a^2}$

$a\lambda^2(a^2\lambda - 1) = 0$

$\rho(B_{GS}) = \frac{1}{a^2} < 1 \qquad a^2 > 1 \quad |a| > 1 \ (a < -1 \cup a > 1)$

$R(B_J) = -\log\frac{1}{|a|} = \cancel{-\log 1} + \log|a| = \log|a|$

$R(B_{GS}) = -\log\frac{1}{a^2} = \cancel{-\log 1} + \boxed{\log a^2 = 2\log|a|} = 2R(B_J)$

ATTENZIONE A QUESTO PASSAGGIO!!!!

Prova Scritta_ Complementi di Matematica _ Crema _

3) 12/9/2012

Stima asintotica $\quad H = \dfrac{b-a}{M}$

$a = 0$

$b = 1$

$$\left|\frac{H^2}{12}\left[f'(a) - f'(b)\right]\right| < 10^{-4}$$

$f(x) = \frac{1}{2} + e^x$

$f'(x) = e^x$

$$\frac{1}{12} \cdot \left(\frac{1}{M}\right)^2 \left| e^0 - e^1 \right| < \frac{1}{10^4}$$

$$\frac{1}{12} \cdot \frac{1}{M^2} \left|(1 - e)\right| < \frac{1}{10^4}$$

$$M^2 > \frac{(e-1) \cdot 10000}{12} \qquad M > 37.84 \qquad \overline{M} = 38$$

Stima classica $\quad f''(t) = e^t$

$$\frac{H^2}{12}(b-a) \underbrace{\max_{a \le t \le b} |f''(t)|}_{e^1} < 10^{-4}$$

$$\frac{1}{12} \cdot \frac{1}{M^2} \cdot e < \frac{1}{10^4}$$

$$M^2 > \frac{10000\, e}{12} \quad \ldots\ldots \quad \overline{M} = 48$$