

$$1) \lambda(B_J) = \left\{ \alpha; \frac{-\alpha - \sqrt{\alpha^2 + 8\alpha}}{2}; \frac{-\alpha + \sqrt{\alpha^2 + 8\alpha}}{2} \right\} \quad \alpha > 0$$

$$|\alpha| < 1 \Rightarrow |-\alpha + \sqrt{\alpha^2 + 8\alpha}| < 2$$

$$\alpha^2 + 8\alpha < (2 + \alpha)^2 \quad 4\alpha < 4 \quad \alpha < 1$$

$$\rightarrow | \alpha + \sqrt{\alpha^2 + 8\alpha} | < 2$$

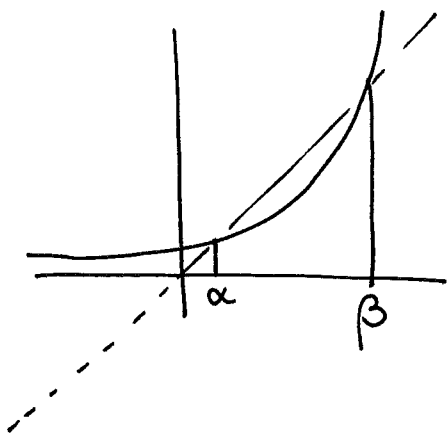
$$\alpha^2 + 8\alpha < 4 - 4\alpha + \alpha^2 \quad \alpha < \frac{1}{3}$$

$$\Rightarrow 0 < \alpha < \frac{1}{3}$$

$$\lambda(B_{GS}) = \{0; 0; \alpha\} \quad \rho(B_{GS}) = |\alpha| < 1$$

$$\begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ 1 & 1 & 1 \\ \frac{1}{4} & \frac{1}{4} & 1 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{4} & \frac{1}{4} & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{3}{4} \\ 0 & 0 & \frac{3}{4} \end{bmatrix}$$

2) •



$$1 < \alpha < 2$$

$$8 < \beta < 9$$

$$g'(x) = \frac{1}{4} e^{x/4}$$

Cond. Suff.
 \uparrow
 $e < 1 \quad x \in [0, 2]$

$$\frac{1}{4} e^{1/4} < g'(\alpha) < \frac{1}{4} e^{1/2}$$

1° ordine

$$0.32$$

$$0.41$$

$$\frac{1}{4} e^2 < g'(\beta) < \frac{1}{4} e^{9/4}$$

$$> 1$$

$$> 1$$

$$x_0 < \beta \quad x_n \rightarrow \alpha; \quad x_0 > \beta \quad x_n \rightarrow +\infty$$

• metodo di Newton $f(x) = x e^{-\frac{x}{4}} - 1$

$$f(0) = -1 \quad f(2) = \frac{2}{\sqrt{e}} - 1 > 0$$

$$f'(x) = e^{-\frac{x}{4}} \left(1 - \frac{x}{4} \right) > 0 \quad x \in [0, 2] \quad f'(0) = 1; f'(2) = \frac{1}{2\sqrt{e}}$$

$$f''(x) = e^{-\frac{x}{4}} \left(\frac{x}{16} - \frac{1}{2} \right) > 0$$

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\Rightarrow Teorema
 $\forall x_0 \in [0, 2]$

$$\left| \frac{1}{1} \right| < 2; \quad \left| \frac{\frac{2 - \sqrt{e}}{\sqrt{e}}}{\frac{1}{2\sqrt{e}}} \right| \approx 0.6 < 2$$

mdN converge a
 $\alpha \in [0, 2]$

3) Per esempio: Trapezzi composti:

$$(b-a) \frac{H^2}{12} \max |f''(t)|$$

$$H = \frac{\pi}{m} \quad b-a = \pi$$

$$f'(x) = \sin x + x \cos x$$

$$f''(x) = \cos x + \cos x - x \sin x = 2 \cos x - x \sin x$$

$$|f''(x)| \leq 2|\cos x| + |x| |\sin x| \leq 2 + \pi$$

$$\frac{\pi}{12} \cdot \left(\frac{\pi}{m}\right)^2 \cdot (2 + \pi) < \frac{1}{100}$$

$$\frac{1}{12} \cdot \pi^3 (2 + \pi) \cdot 100 < m^2 \quad m > 36.449 \Rightarrow \bar{m} = 37$$

4) $S(1^-) = S(1^+) \wedge S'(1^-) = S'(1^+) \wedge S''(1^-) = S''(1^+) \wedge$
 $S''(0) = S''(2) = 0$. Si ottiene:

$$a = 2; \quad b = -1; \quad c = -3; \quad d = 1$$

$$\text{(dove: } S'(x) = \begin{cases} 2 - 3x^2 \dots \\ b + 2c(x-1) + 3d(x-1)^2 \dots \end{cases}$$

$$S''(x) = \begin{cases} -6x \dots \\ 2c + 6d(x-1) \dots \end{cases}$$

5) Vedi dispensa E, Z.