CALCOLO NUMERICO 1 (24 febbraio 2011)

CORREZIONE

ESERCIZIO 1

Si tenga presente che $\alpha \in (0,1)$.

1.1) $\det(A) = (1-\alpha^5)^2$; $\|A\|_\infty = \max\{1+\alpha,\ 1+\alpha^2,\ 1+\alpha^3,\ 1+\alpha^4\} = 1+\alpha$.

1.2) $\det\begin{pmatrix} \lambda & 0 & 0 & \alpha \\ 0 & \lambda & \alpha^2 & 0 \\ 0 & \alpha^3 & \lambda & 0 \\ \alpha^4 & 0 & 0 & \lambda \end{pmatrix} = (\lambda^2 - \alpha^5)^2 = 0,\ \lambda = \pm\sqrt{\alpha^5},\ \rho(B_J) = \sqrt{\alpha^5}.$

$\det\begin{pmatrix} \lambda & 0 & 0 & \alpha \\ 0 & \lambda & \alpha^2 & 0 \\ 0 & \lambda\alpha^3 & \lambda & 0 \\ \lambda\alpha^4 & 0 & 0 & \lambda \end{pmatrix} = (\lambda^2 - \lambda\alpha^5)^2 = 0,\ \lambda = \alpha^5,\ \lambda = 0,\ \rho(B_{GS}) = \alpha^5.$

$\rho(B_{GS}) = \rho(B_J)^2 < 1$ essendo $\alpha \in (0,1)$.

$R(B_{GS}) = -\ln \rho(B_{GS}) = -\ln \rho(B_J)^2 = 2R(B_J)$.

1.3) $\alpha = \dfrac{1}{2}, \quad \rho(B_{GS}) = \dfrac{1}{32}, \quad k \geq \dfrac{-\ln 10^{-4}}{-\ln \rho(B_{GS})},\ \dfrac{-\ln 10^{-4}}{-\ln \frac{1}{32}} \approx 2.6575 \Rightarrow \overline{k} = 3.$

ESERCIZIO 2

Tenendo conto che $x \in (1,\infty)$, si ha:

$$K(f(x)) = \left|\frac{xf'(x)}{f(x)}\right| = \left|\frac{x(\sqrt{x-1}+\sqrt{x+1})}{2\sqrt{x+1}\sqrt{x-1}}\ \frac{1}{\sqrt{x+1}+\sqrt{x-1}}\right| = \frac{x}{2\sqrt{x+1}\sqrt{x-1}};$$

$$\frac{x}{2\sqrt{x+1}\sqrt{x-1}} < 1,\ \Rightarrow 3x^2 > 4,\ \Rightarrow x > \frac{2}{\sqrt{3}} \approx 1.1547.$$

ESERCIZIO 3

$x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)} = x_k + 2\sqrt{x_k}\,(1-\sqrt{x_k}) = 2\sqrt{x_k} - x_k.$

Studio di $y = g(x) \equiv 2\sqrt{x} - x$; $(\beta = 0,\ \alpha = 1)$; C.E. $x \geq 0$; $g(x) \geq 0$ se $0 \leq x \leq 4$.

$g'(x) = \dfrac{1}{\sqrt{x}} - 1 \geq 0$, se $0 < x \leq 1$; Massimo: $(1,1)$.

Studio della convergenza al variare di $x_0 > 0$:

$\alpha < x_0 < 4 \longrightarrow 0 < x_1 < \alpha$;

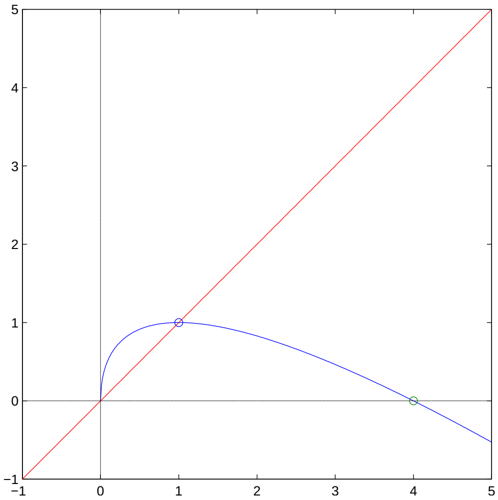
$0 < x_0 < \alpha \longrightarrow x_n \nearrow \alpha$ (succ. monotona cresc. lim. sup. da α)

$x_0 > 4 \longrightarrow x_1 < 0$: STOP.

$x_0 = 4 \longrightarrow x_1 = 0 = \beta$.

$g'(1) = 0$ (ovvio per il metodo di Newton, $\alpha = 1$ ha molteplicità 1)

$g''(1) \neq 0 \Longrightarrow$ ordine 2.



ESERCIZIO 4

x_i	0	1	2	3	4
x_i^2	0	1	4	9	16
$y_i = f(x_i)$	0	2	2	2	4
$x_i y_i$	0	2	4	6	16
$p_1(x_i)$	$\frac{2}{5}$	$\frac{6}{5}$	2	$\frac{14}{5}$	$\frac{18}{5}$
$\lvert y_i - p_1(x_i)\rvert$	$\frac{2}{5}$	$\frac{4}{5}$	0	$\frac{4}{5}$	$\frac{2}{5}$
$\lvert y_i - p_1(x_i)\rvert^2$	$\frac{4}{25}$	$\frac{16}{25}$	0	$\frac{16}{25}$	$\frac{4}{25}$

$$\sum_{i=0}^{4} x_i = 10;\ \sum_{i=0}^{4} x_i^2 = 30;\ \sum_{i=0}^{4} y_i = 10;\ \sum_{i=0}^{4} x_i y_i = 28.$$

$$5c_0+10c_1 = 10,\ \ 10c_0+30c_1 = 28 \implies c_0 = \frac{2}{5},\ c_1 = \frac{4}{5},\ p_1(x) = \frac{2}{5}+\frac{4}{5}x.$$

$$M_x = 2,\ M_y = 2,\ p_1(2) = \frac{2}{5}+\frac{4}{5}\cdot 2 = 2;\ \ \sum_{i=0}^{4} |y_i - p_1(x_i)|^2 = \frac{8}{5}.$$