Calcolo Numerico 1 (Matematica_Milano) 2 febbraio 2012

) $\lambda(B_{GS}) = \{0, 0, 0, \frac{3}{4}\}$ $\quad \rho(B_{GS}) = \frac{3}{4} < 1$ Converge

2) A simmetrica, $a_{ii} > 0$ GS converge $\Leftrightarrow$ A def. pos.

..3)

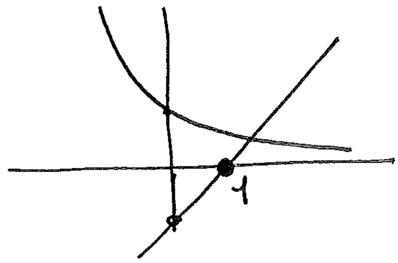
$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$$

1.4) $B_{LU} = I - \alpha U \qquad \lambda(B_{LU}) = \{1-\alpha, 1-\alpha, 1-\alpha, 1-\frac{\alpha}{4}\}$

$|1-\alpha| < 1 \Leftrightarrow 0 < \alpha < 2 \quad \Big| \quad |1 - \frac{\alpha}{4}| < 1 \Leftrightarrow 0 < \alpha < 8 \quad \Rightarrow \boxed{0 < \alpha < 2}$

2) Vedi file a parte.

$e^{-x} = x - 1 \qquad f(x) = e^{-x} - x + 1$




- $f(1) = e^{-1} > 0 \qquad f(2) = e^{-2} - 1 < 0$
- $f'(x) = -e^{-x} - 1 < 0 \quad \forall x$
- $f''(x) = e^{-x} > 0 \quad \forall x$

$f'(1) = -e^{-1} - 1 \qquad f'(2) = -e^{-2} - 1$

$$\left| \frac{\frac{1}{e}}{\frac{1}{e} + 1} \right| < 1 \text{ sì} \qquad \left| \frac{-\frac{1}{e^2} + 1}{\frac{1}{e^2} + 1} \right| = \frac{1 - \frac{1}{e^2}}{1 + \frac{1}{e^2}} < 1 \text{ sì}$$

La soluzione $\alpha$ t.c. $f(\alpha) = 0$ $\exists$ ed è unica, $\in (1, 2)$ e mdN converge ad $\alpha$ $\forall x_0 \in (1, 2)$

4) $\int_{-1}^{1} f(x)\,dx \approx \omega_1 f(\alpha_1) + \omega_2 f(\alpha_2)$

$r=0$, $f=1$: $\int_{-1}^{1} f(x)\,dx = 2$ $\qquad \omega_1 + \omega_2 = 2$

$r=1$, $f=x$: $\int_{-1}^{1} f(x)\,dx = 0$ $\qquad \omega_1\alpha_1 + \omega_2\alpha_2 = 0$

$r=2$, $f=x^2$: $\int_{-1}^{1} f(x)\,dx = \frac{2}{3}$ $\qquad \omega_1\alpha_1^2 + \omega_2\alpha_2^2 = \frac{2}{3}$

$r=3$, $f=x^3$: $\int_{-1}^{1} f(x)\,dx = 0$ $\qquad \omega_1\alpha_1^3 + \alpha_2\alpha_2^3 = 0$

$$\Downarrow$$

$\omega_1 = \omega_2 = 1 \qquad \alpha_1 = -\frac{1}{\sqrt{3}} \qquad \alpha_2 = \frac{1}{\sqrt{3}}$

Formula di Gauss-Legendre a 2 punti

Per esempio

$f(x) = x^4$

$$\int_{-1}^{1} x^4\,dx = \frac{2}{5}$$

$$f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$$

5) $K_f(x) = \left| \dfrac{x\,(-2x)\,e^{-x^2}}{e^{-x^2}} \right| = 2x^2$