Calcolo Numerico 1 - Matematica M1 - 16/2/2012

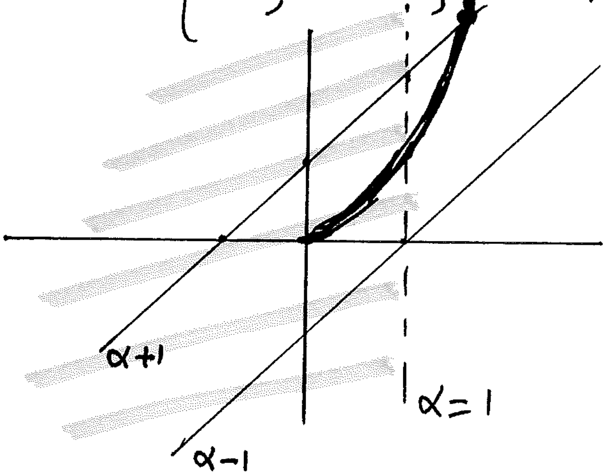
1.1) D.P. $\begin{cases} \alpha > 0 \\ \alpha^3 > 0 \\ \alpha^2(\alpha^2-1) > 0 \end{cases} \iff \alpha > 1$

$\Downarrow$

$\|A\|_2 = \rho(A) = \max\{\alpha^2, |\alpha-1|, |\alpha+1|\} = \max\{\alpha^2, \alpha-1, \alpha-1\}$

$\alpha > 1$

$\lambda(A) = \{\alpha^2, \alpha \pm 1\}$



$\alpha^2 = \alpha+1 \qquad \alpha^2 - \alpha - 1 = 0 \qquad \alpha = \frac{1 \pm \sqrt{5}}{2} \Rightarrow \rho(A) = \begin{cases} \alpha^2 & \text{se } \alpha > \frac{1+\sqrt{5}}{2} \\ \alpha+1 & \text{se } 1 < \alpha < \frac{1+\sqrt{5}}{2} \end{cases}$

1.2) $\lambda(B_{GS}) = \{0, 0, \frac{1}{\alpha^2}\} \Rightarrow \rho(B_{GS}) = \frac{1}{\alpha^2} > 0 \iff |\alpha| > 1$

1.3) $L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 4 & 0 \\ 0 & 0 & \frac{3}{2} \end{pmatrix}$

$\|L\|_\infty = \frac{3}{2}$

$\|U\|_\infty = 4$

$\|A\|_\infty = 4$

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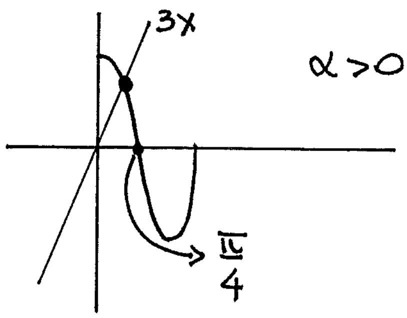
2) • $g(x) = \frac{\cos 2x}{3} \qquad g'(x) = -\frac{2}{3} \operatorname{sen} 2x \qquad |g'(x)| \le \frac{2}{3}$

$\Rightarrow$ convergenza

$g'(\alpha) \neq 0 \quad (\alpha \neq 0)(\alpha \neq \frac{\pi}{2})$

$|g'(\alpha)| < 1 \Rightarrow 1^\circ$ ordine



$\alpha > 0 \qquad \alpha < \frac{\pi}{4}$

• $g(x) = x - \frac{3x - \cos 2x}{3 + 2\sin 2x}$ è il metodo di Newton

applicato a $f(x) = 3x - \cos 2x$

$f(x) = 3x - \cos 2x \qquad \left[0, \frac{\pi}{4}\right]$

- $f(0) = -1 \qquad f\left(\frac{\pi}{4}\right) = \frac{3\pi}{4}$
- $f'(x) = 3 + 2 \,\text{sen}\, 2x > 0 \qquad x \in \left[0, \frac{\pi}{4}\right]$
- $f''(x) = 4 \cos 2x > 0 \qquad$ "

$f'(0) = 3 \qquad f'\left(\frac{\pi}{4}\right) = 5$

$\left| \frac{1}{3} \right| < \frac{\pi}{4} \qquad \left| \frac{\frac{3\pi}{4}}{5} \right| = \frac{3\pi}{20} < \frac{\pi}{4}$

$\exists!\, \alpha$ t.c. $f(\alpha) = 0$; m.d. N converge a α
$\alpha \in \left[0, \frac{\pi}{4}\right] \qquad \forall\, x_0 \in \left[0, \frac{\pi}{4}\right]$

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3)

x_i	0	1	$\frac{3}{2}$	2	3
y_i	0	$\frac{1}{e}$	$\frac{3}{2}\frac{1}{e\sqrt{e}}$	$\frac{2}{e^2}$	$\frac{3}{e^3}$

$y = f(x) = x e^{-x}$

Costruire, $\forall\, [x_{i-1}, x_i]$ la retta interpolante
$(x_{i-1}, y_{i-1}) \quad (x_i, y_i) \ldots$

Stima: $\frac{1}{8} h^2 \max_{[0,3]} |f''(x)|$

$h = h_{max} = 1$

$f'(x) = e^{-x} - x e^{-x} = (1-x) e^{-x}$

$f''(x) = -1 \cdot e^{-x} + (1-x)(-1) e^{-x} = e^{-x}(x-2)$

$\max_{[0,3]} e^{-x} |x-2| \leq 1 \cdot 2$

$\max_{[0,3]} |f(t) - S_1(t)| \leq \frac{1}{8} 1 \cdot 2 = \frac{1}{4}$

4) Si veda dispensa E.Z.
(Diario delle lezioni....)

Metodi di Richardson (M. del gradiente)
in particolare l'Esercizio p. 62–65

5) $K_f(x) = \dfrac{x}{\sqrt{2x+1}\,\sqrt{2x-1}} < 10$

C.E. $x > \frac{1}{2}$

$x > \dfrac{10}{\sqrt{399}} = \frac{1}{2} + \text{"}\varepsilon\text{"}$