

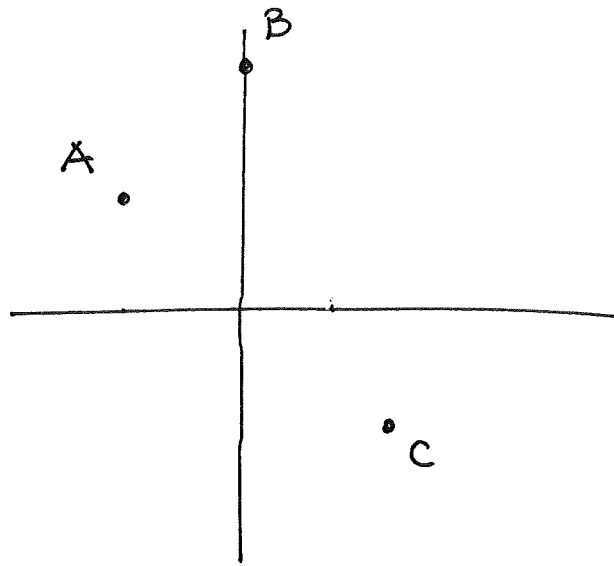
3) Costruire la spline cubica interpolante naturale
 i punti $(-1, 1)$ $(0, 2)$ $(1, -1)$

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$$A(-1; 1)$$

$$B(0; 2)$$

$$C(1; -1)$$



$$S(x) = \begin{cases} a(x+1)^3 + b(x+1)^2 + c(x+1) + d & -1 \leq x < 0 \\ ex^3 + fx^2 + gx + h & 0 \leq x \leq 1 \end{cases}$$

$$S'(x) = \begin{cases} 3a(x+1)^2 + 2b(x+1) + c & -1 \leq x < 0 \\ 3ex^2 + 2fx + g & 0 \leq x \leq 1 \end{cases}$$

$$S''(x) = \begin{cases} 6a(x+1) + 2b & -1 \leq x < 0 \\ 6ex + 2f & 0 \leq x \leq 1 \end{cases}$$

Si impongono le condizioni:

- interpolazione 3 condizioni
- $S''(-1) = S''(1) = 0$ 2 "
- $S \in C^2[-1, 1]$ 3 "

$$\left. \begin{array}{ll}
 S(-1) = 1 & d = 1 \\
 S(0^-) = 2 & a + b + c + d = 2 \\
 S(0^+) = 2 & h = 2 \\
 S(1) = -1 & e + f + g + h = -1 \\
 S''(-1) = 0 & 2b = 0 \\
 S''(1) = 0 & 6e + 2f = 0 \\
 S'(0^-) = S'(0^+) & 3a + 2b + c = g \\
 S''(0^-) = S''(0^+) & 6a + 2b = 2f
 \end{array} \right\} \Rightarrow \begin{array}{l}
 \boxed{d = 1} \\
 a + c = 1 \\
 \Rightarrow \boxed{h = 2} \\
 e + f + g = -3 \\
 \Rightarrow \boxed{b = 0} \\
 3e + f = 0 \\
 3a + c = g \\
 6a = 2f \\
 3a = f
 \end{array}$$

$$\left\{ \begin{array}{l}
 a + c = 1 \\
 e + f + g = -3 \\
 3e + f = 0 \\
 3a + c - g = 0 \\
 3a - f = 0
 \end{array} \right. \Rightarrow \left\{ \begin{array}{l}
 \boxed{e = -\frac{f}{3}} \\
 \boxed{a = \frac{f}{3}} \\
 \frac{f}{3} + c = 1 \\
 -\frac{f}{3} + f + g = -3 \\
 f + c - g = 0
 \end{array} \right.$$

$$\left\{ \begin{array}{l}
 \boxed{c = 1 - \frac{f}{3}} \\
 \boxed{g = -3 - \frac{2}{3}f} \\
 \boxed{f + c - g = 0}
 \end{array} \right. \Rightarrow \begin{array}{l}
 f + \overbrace{1 - \frac{f}{3}}^{+c} + \overbrace{3 + \frac{2}{3}f}^{-g} = 0 \\
 \frac{4}{3}f = -4 \quad \boxed{f = -3}
 \end{array}$$

$$c = 2$$

$$g = -1$$

\Downarrow

$$w = -1, \quad e = 1$$

$$S(x) = \begin{cases} -(x+1)^3 + 2(x+1) + 1 \\ x^3 - 3x^2 - x + 2 \end{cases} = \begin{cases} -x^3 - 3x^2 - x + 2 \\ x^3 - 3x^2 - x + 2 \end{cases}$$