CALCOLO NUMERICO 1 _ MATEMATICA MI _ 19.06.12

1) $\det A = -\alpha(2\alpha^2+\alpha-1)$ A non singolare $\Leftrightarrow \alpha \neq \frac{1}{2} \wedge \alpha \neq -1$ $(\alpha \neq 0)$

$$B_J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2\alpha} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -\alpha & \alpha \\ -\alpha & 0 & -\alpha \\ \alpha & -\alpha & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\alpha & \alpha \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ \alpha & -\alpha & 0 \end{bmatrix}$$

$$\|B_J\|_1 = \max\left\{\frac{1}{2}+|\alpha|; 2|\alpha|\right\} = \begin{cases} 2|\alpha|, & |\alpha| > \frac{1}{2} \\ \frac{1}{2}+|\alpha|, & |\alpha| \leq \frac{1}{2} \end{cases}$$

$\rho(B_J) \leq \|B_J\|_1 < 1$ per $\alpha \in (0, \frac{1}{2})$

oppure si calcolano gli autovalori di B_J

$\lambda(B_J) = \left\{\alpha; \frac{\alpha \pm \sqrt{\alpha^2+4\alpha}}{2}\right\}$. Si verifica $|\lambda_i| < 1$ per $\alpha \in (0; \frac{1}{2})$

2) $\frac{b-a}{2^n} = \frac{3}{2^n} < 2^{-10}$ $\quad n-10 > \frac{\log 3}{\log 2}$, $n > 10 + \frac{\log 3}{\log 2}$, $\bar{n} = 12$

$f(-2) = -43$; $f(1) = 11$

$f'(x) = 3(x^2-2x+4) > 0 \ \forall x$, $f'(-2) = 36$, $f'(1) = 9$

$f''(x) = 6x-6 \geq 0 \quad x \geq 1$ OK

$\left|\frac{43}{36}\right| < 3$; $\left|\frac{11}{9}\right| < 3$ $\quad \exists! \alpha \in (-2;1)$ t.c. $f(\alpha) = 0$

mdN converge ad α $\forall x_0 \in (-2;1)$.

3) $H = \frac{b-a}{M}$ $\quad |err| \leq \frac{H^2}{12}(b-a) \max\limits_{0 \leq x \leq 2} |f''(x)|$

$f'(x) = -2x e^{-x^2}$

$f''(x) = 2(2x^2-1)e^{-x^2}$ $\quad |f''(x)| \leq 2 \cdot 7 \cdot 1 = 14$

oppure $f'''(x)$.... calcolo max di $|f''(x)|$ studiando il segno di $f'''(x)$ e $|f''(0)|$, $|f''(2)|$.... $\Rightarrow \max|f''(x)| = \frac{4}{e^{3/2}}$

$\left(\frac{2}{M}\right)^2 \cdot \frac{1}{\cancel{12}_{\cancel{6}_3}} \cdot \cancel{2} \cdot \cancel{14}^{7} \leq 10^{-3}$

$\frac{4 \cdot 7}{3} 10^3 \leq M^2$ $\quad M > \sqrt{9333}$

$\bar{M} = 97$

$\left(\frac{2}{M}\right)^2 \cdot \frac{2}{12} \cdot \frac{4}{e^{3/2}} < 10^{-3}$

$M^2 > 595$ $\quad \bar{M} = 25$

4) $\sum x_i = 4$; $\sum x_i^2 = 10$; $\sum y_i = 2+k$; $\sum x_i y_i = 3k$

$r(x) = c_0 + c_1 x \qquad c_0 = \frac{10-k}{7}; \quad c_1 = \frac{5k-8}{14}$

Allineamento: per esempio $p_1(0) = 2 \quad \frac{10-k}{7} = 2 \quad k = -4$

$(\Rightarrow p(3) = k;\ p(1) = 0)$

5) $F(c) = \int_a^b [f(x)]^2 dx - 2c \int_a^b f(x)\,dx + c^2 \int_a^b 1 \cdot dx$

$\frac{dF(c)}{dc}$ ..... parabola .....

(Ascissa del vertice) $\quad c = \frac{\int_a^b f(x)\,dx}{b-a}$

Valor medio dell'integrale