Calcolo Numerico 1 _ Matematica MILANO _ 12/07/2012

1) 1.1) $\det A = (a-2)(a+1)$ A non sing $\Leftrightarrow a \neq 2 \wedge a \neq 1$

1.2) $\left[|a| > 2 \wedge |1-a| > 1 \right] \Rightarrow |a| > 2$

1.3) $\lambda(B_J) = \pm \sqrt{\frac{2}{a(a-1)}}$ se $a < 0 \vee a > 1$

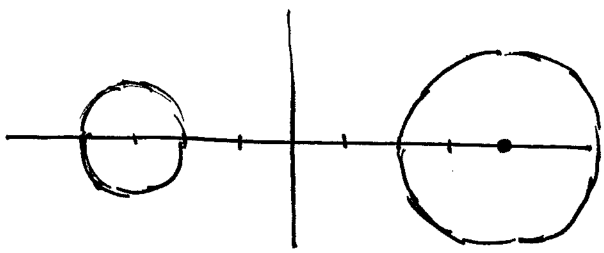
$= \pm i \sqrt{\frac{2}{-a(a-1)}}$ se $0 < a < 1$

$\rho(B_J) = \sqrt{\frac{2}{|a(a-1)|}} < 1$ $\quad a < -1 \vee a > 2$

1.4) La matrice è tridiagonale $\Rightarrow a < -1 \vee a > 2$

Se $a = 4$ $\quad A = \begin{bmatrix} 4 & 2 \\ -1 & -3 \end{bmatrix}$

$|\lambda - 4| \leq 2 \cup |\lambda + 3| \leq 1$



Cerchi disgiunti $\Rightarrow \lambda_1 \in Z_1$ e $\lambda_2 \in Z_2 \Rightarrow \lambda_i \in \mathbb{R}$

2) $L_0(x) = \frac{x(x+1)}{2}$; $L_1(x) = 1 - x^2$; $L_2(x) = \frac{x(x-1)}{2}$

$p_2(x) = 2x^2 - 1$

$n = 4$ $\quad \sum x_i = 0$; $\sum x_i^2 = 3$; $\sum y_i = 1$; $\sum x_i y_i = 0$

$r(x) = c_0 + c_1 x \Rightarrow r(x) = \frac{1}{5}$

Generalizzazione:

$$\sum_{k=0}^{n} x_k = 0 \qquad \sum_{k=0}^{n} x_k y_k = 0 \qquad \forall n$$

$$r(x) = \frac{1}{n+1}$$

3) $r=0 \quad \omega_1+\omega_2=1$

$r=1 \quad \int_{-1}^{1} x\,dx = FQ(x) \quad \forall\, \omega_1, \omega_2, \alpha$

$r=2 \quad \omega_1+\omega_2\alpha^2 = \frac{1}{3}$

$\Rightarrow \quad \omega_1 = \frac{1-3\alpha^2}{3(1-\alpha^2)} \qquad \omega_2 = \frac{2}{3(1-\alpha^2)} \qquad (*)$

$r=3$ (vedi $r=1$, $f=x^3$)

$r=4 \quad \omega_1+\omega_2\alpha^4 = \frac{1}{5}$

Sostituendo i valori $(*)$ si ha

$5\alpha^4-6\alpha^2+1=0 \quad (\alpha^2=1 \text{ NO}) \quad \alpha = \frac{1}{\sqrt{5}} \quad (\alpha>0)$

4) $g(1)=1 \quad (1+\cancel{\beta})+\cancel{\gamma}-\cancel{\beta}-\cancel{\gamma}=1$

- $g(x)=x+\beta x^3+\gamma e^{x-1}-(\beta+\gamma)$
- $g'(x)=1+3\beta x^2+\gamma e^{x-1} \qquad g'(1)=\boxed{1+3\beta+\gamma=0}$
- $g''(x)=6\beta x+\gamma e^{x-1}$

  $g''(1)=6\beta+\gamma$

  $\begin{cases} 6\beta+\gamma=0 \\ 1+3\beta+\gamma=0 \end{cases} \quad \cdots\cdots \quad \begin{cases} \beta=\frac{1}{3} \\ \gamma=-2 \end{cases} \quad [*]$

- $g'''(x)=6\beta+\gamma e^{x-1}$

  $g'''(1)=6\beta+\gamma$

  Sostituendo $[*]$ $\quad 6\cdot\frac{1}{3}-2=0$

  Il metodo ha ordine 4