Seconda prova in itinere

1)
$$A = \begin{bmatrix} 1 & \alpha & \alpha^2 & \dots & \alpha^n \\ \alpha & 1 & & & \\ \vdots & & 1 & & \\ \vdots & & & \ddots & \\ \alpha^n & & & & 1 \end{bmatrix}$$

$n \geq 3$

$-\frac{1}{2} < \alpha < \frac{1}{2}$

A simmetrica $\Rightarrow \lambda \in \mathbb{R}$

Intervalli di Gershgorin

A) $|\lambda - 1| \leq |\alpha| + |\alpha|^2 + |\alpha|^3 + \dots + |\alpha|^n$

B) $|\lambda - 1| \leq |\alpha|$

$\vdots$

C) $|\lambda - 1| \leq |\alpha|^n$

$$A) \cup B) \cup C) \Rightarrow 1 - \sum_{K=1}^{n} |\alpha|^n \leq \lambda \leq 1 + \sum_{K=1}^{n} |\alpha|^n$$

$$\sum_{K=1}^{n} |\alpha|^n < \sum_{K=1}^{\infty} |\alpha|^n = \frac{|\alpha|}{1 - |\alpha|} = 1 \Rightarrow 1 - \sum_{K=1}^{n} |\alpha|^n > 0$$

$$\lambda \leq \frac{1 + \sum_{K=1}^{n} |\alpha|^n}{1 - \sum_{K=1}^{n} |\alpha|^n}$$

$$\|A\|_\infty = \max\left\{1 + |\alpha| + |\alpha|^2 + \dots + |\alpha|^m ; 1 + |\alpha| ; \dots ; 1 + |\alpha|^n\right\} =$$
$$= \sum_{K=0}^{m} |\alpha|^K$$

$\alpha = 1/3 \quad n = 5$

$$1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^5 = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} = \frac{364}{243}$$

2ª prova in itinere

2)
$$A = \begin{bmatrix} 4 & -1 & & & 1-\frac{4}{n} \\ -1 & 4 & -1 & 0 & \\ & -1 & \ddots & & \\ & & \ddots & & -1 \\ 1-\frac{4}{n} & & & -1 & 4 \end{bmatrix} \qquad n \geq 3$$

E' simmetrica $\lambda \in \mathbb{R} \Rightarrow$ Teorema Gershgorin

Teorema: $|\lambda - 4| \leq 1 + \left|1 - \frac{4}{n}\right|$ | $3 - \left|1-\frac{4}{n}\right| \leq \lambda \leq 5 + \left|1 - \frac{4}{n}\right|$

$|\lambda - 4| \leq 1 + 1$ | $2 \leq \lambda \leq 6$

$$\min \lambda \geq 3 - \left|1 - \frac{4}{n}\right| \overset{?}{>} 0$$

$$\left|1 - \frac{4}{n}\right| < 3 \qquad \begin{cases} 1 - \frac{4}{n} < 3 \\ 1 - \frac{4}{n} > -3 \end{cases} \qquad \begin{cases} 2 > -\frac{4}{n} & \text{ovvio} \\ 4 > \frac{4}{n} & \text{sì} \end{cases}$$

$\lambda \in \mathbb{R}^+$

$\Rightarrow$ Simmetrica, $\lambda \in \mathbb{R}^+ \Rightarrow$ Definita positiva

$$B_J = D^{-1}(E+F)$$

$$B_J = \begin{bmatrix} \frac{1}{4} & & & \\ & \frac{1}{4} & & \\ & & \ddots & \\ & & & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 & 1 & & \frac{4}{n}-1 \\ 1 & 0 & 1 & \\ & 1 & \ddots & 1 \\ \frac{4}{n}-1 & & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & & & \frac{1}{n}-\frac{1}{4} \\ \frac{1}{4} & 0 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \frac{1}{4} \\ \frac{1}{n}-\frac{1}{4} & & & \frac{1}{4} & 0 \end{bmatrix}$$

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2^a prova in itinere

$$\|B_J\|_\infty = \max\left\{ \frac{1}{4} + \left|\frac{1}{n} - \frac{1}{4}\right| ; \frac{1}{2} \right\}$$

Controllo: $\frac{1}{4} + \left|\frac{1}{n} - \frac{1}{4}\right| < 1$

$$\left|\frac{1}{n} - \frac{1}{4}\right| < \frac{3}{4} \qquad \begin{cases} \frac{1}{n} - \frac{1}{4} < \frac{3}{4} \\ \frac{1}{n} - \frac{1}{4} > -\frac{3}{4} \end{cases} \qquad \begin{cases} \frac{1}{n} < 1 \\ \frac{1}{n} > -\frac{1}{2} \end{cases} \qquad \text{sì}$$

E' verif. la condizione sufficiente $\Rightarrow$ il metodo di Jacobi è convergente

3) $\sin x = 1 - x$

3.1) $f(x) = \sin x - 1 + x$

$f(0) = -1 \qquad f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} - 1 + \frac{\pi}{3} > 0 \qquad (\approx 0.91)$

$f'(x) = \cos x + 1 \quad > 0 \qquad x \in \left[0 ; \frac{\pi}{3}\right]$

$\exists!\, \alpha \in \left[0, \frac{\pi}{3}\right]$ t.c. $f(\alpha) = 0$

3.2) $$x_{n+1} = x_n - \frac{\sin x_n - 1 + x_n}{\cos x_n + 1}$$

3.3) Controllo le condizioni:

- $f''(x) \geq 0$ oppure ≤ 0 $\quad f''(x) = -\sin x < 0 \quad x \in [0, \frac{\pi}{3}]$

- $\left| \frac{f(0)}{f'(0)} \right| = \frac{1}{2} < \frac{\pi}{3}$

$$\left| \frac{f(\pi/3)}{f'(\frac{\pi}{3})} \right| \cong \left| \frac{0.91}{\frac{3}{2}} \right| \cong 0{,}61 < \frac{\pi}{3}$$

Il metodo converge ad α (unica radice $\in [0, \frac{\pi}{3})$ punto 3.1) $\forall x_0 \in [0, \frac{\pi}{3}]$.